Econ 2450B, Topic 3: Commodities and Public Goods with Redistributive Concerns[1]

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[1] I want to thank Raj Chetty for sharing his slides on public goods, which form the basis for Section 3 of this lecture.

Recap of Topics 1 and 2

- Suppose we have a policy that spends more on G targeted towards those earning around $\$y$ of income
- Need to calculate:
  - Individuals WTP out of their own income for additional G, $s\left(y\right) = \frac{\frac{\partial u_i}{\partial G}}{\lambda_i} = \frac{u_G}{u_c}$
    - (assume homogenous WTP conditional on income)
  - Total cost to the government inclusive of fiscal externalities
    - $1 + FE_G = \frac{d}{dG}\left[q\right]$, where q is the aggregate govt budget
  - Construct MVPF for each individual with earnings y

$$MVPF\left(y\right) = \frac{s\left(y\right)}{1 + FE_G}$$

Recap: Aggregation

- Aggregate using either:
- [SWF] Social marginal utilities of income, $\int \eta(y)\, MVPF(y)$
- [Kaldor-Hicks/Kaplow/Mirrlees 1976] Marginal cost of redistributing to those with income y, $1 + FE(y)$

$$W = \int (1 + FE(y))\, MVPF(y)$$

  - Implicitly compare efficiency of G to efficiency of redistribution through modifications to tax schedule, $T(y)$

Key Difficulty: Estimating FE...

- Implementing these formulae require estimating two fiscal externalities:
  - Impact of G on tax revenue, FE_G
  - Impact of tax changes to those earning y on tax revenue, $FE(y)$, for all y
- Why are these difficult?
  - Dynamics (impact on tax revenue in 30 years...)
  - Bases (impact of income tax changes on capital taxes, sales taxes, food stamp participation, etc...)
  - And, need rich variation in tax policies to identify $FE(y)$ for all y
    - Made progress in Topic 2 by assuming constant taxable income elasticity/etc.
- This lecture: potentially able to ignore all behavioral responses
  - Literature on optimal commodity taxation and optimal public goods
  - Key (weak?) assumption reduces these empirical requirements: "weak separability"

Basic Idea

- Begin with a roadmap of the basic idea
- Many economic models imply a relationship between FE_G and $FE\left(y\right)$
- The social benefit of \$1 of spending on G is given by:

$$W = \int \left(1 + FE\left(y\right)\right) s\left(y\right) dy$$

- Cost is given by $1 + FE_G$
- So, additional spending can increase welfare if and only if

$$\int \left(1 + FE\left(y\right)\right) s\left(y\right) dy \geq 1 + FE_G$$

or

$$\underbrace{\int s\left(y\right) dy}_{\text{Aggregate Surplus}} \geq \int s\left(y\right) FE\left(y\right) dy - FE_G$$

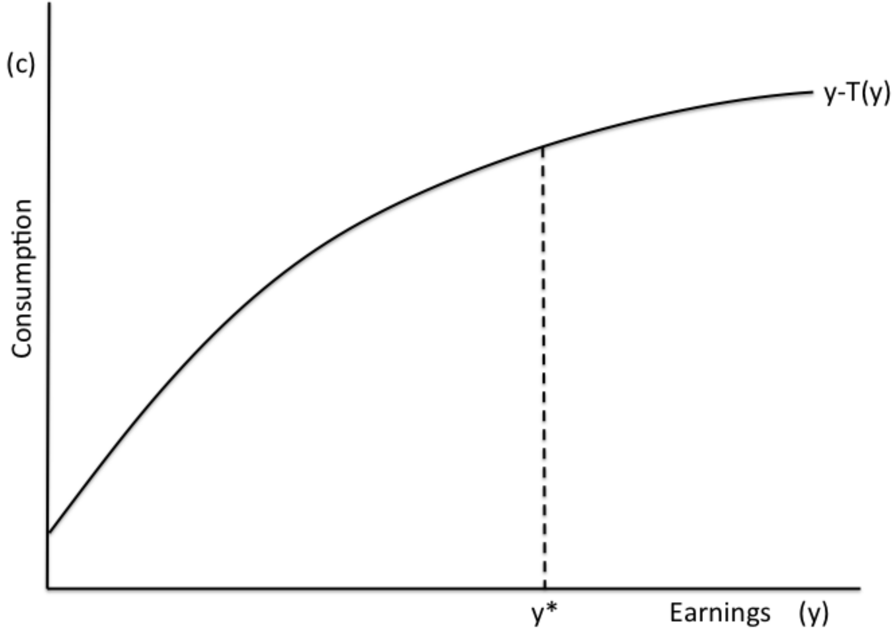
Key Insight

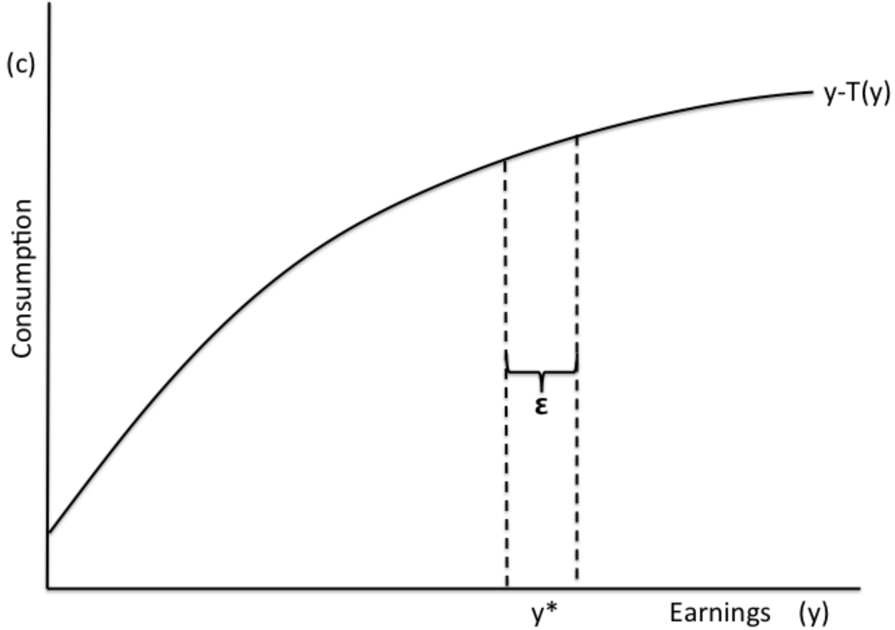
- Key insight: In many cases, reasonable to think that

$$\int s(y) \, FE(y) \, dy = FE_G$$

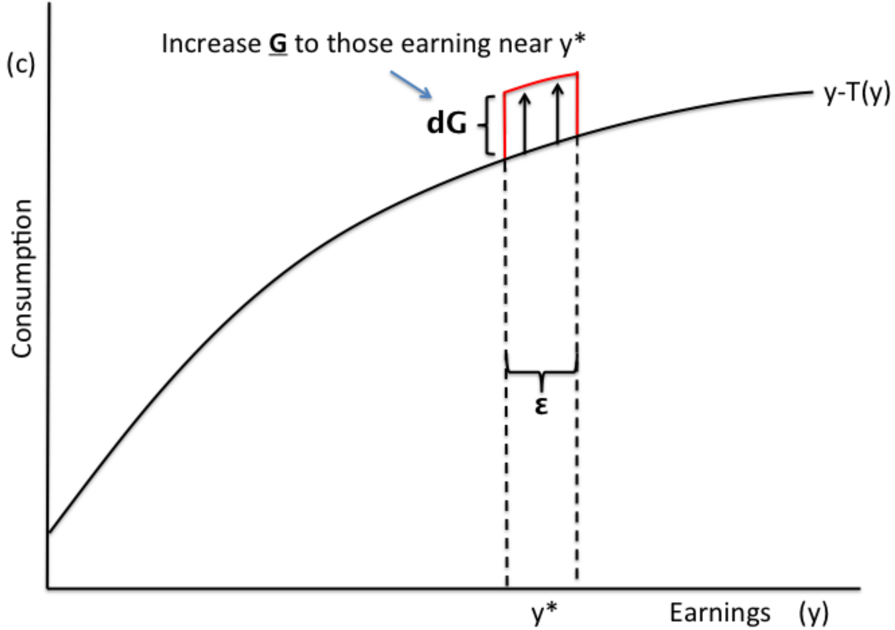
  - Why?

(c)
Consumption
y-T(y)
Earnings (y)

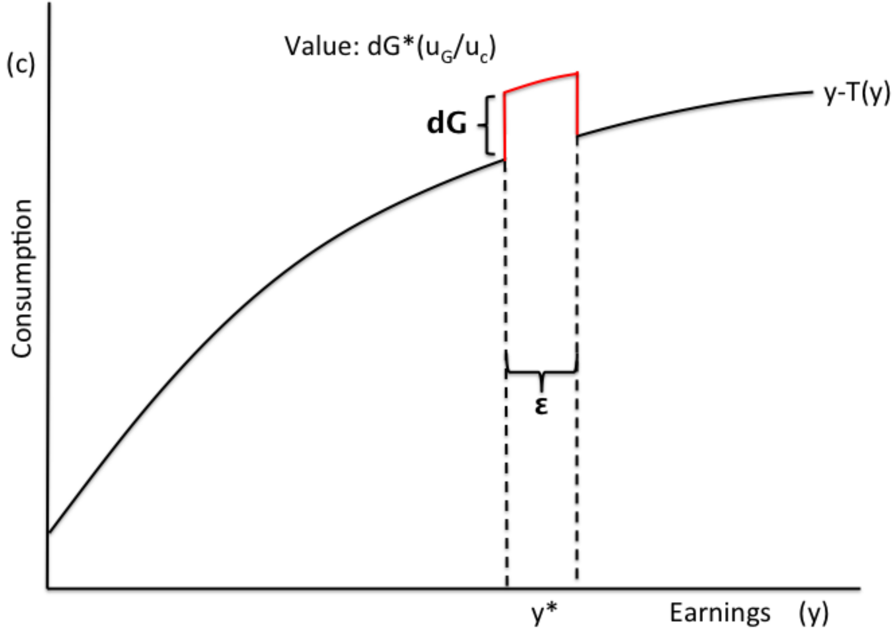

(c)
Consumption
y-T(y)
y*
Earnings (y)



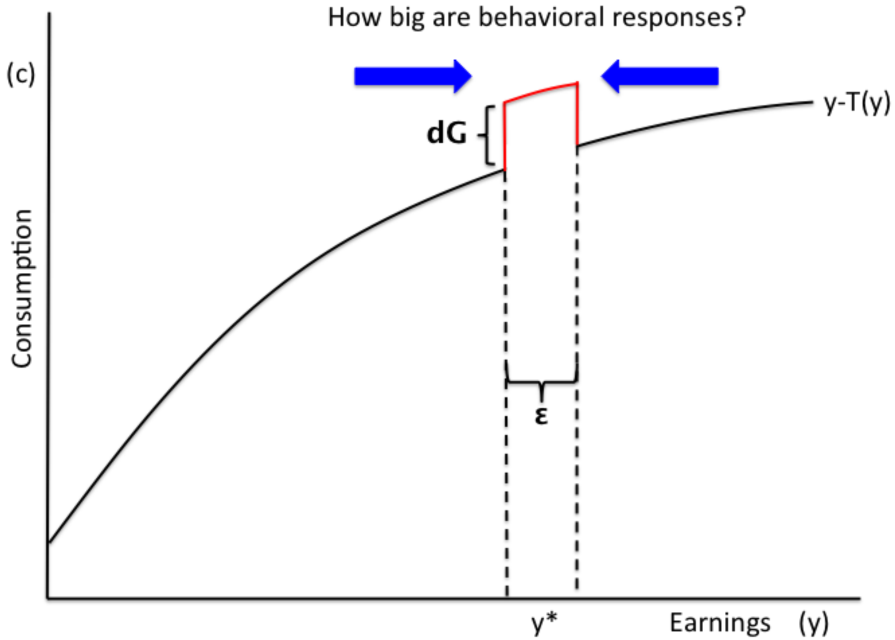
(c)
Consumption
y-T(y)
ε
y*
Earnings (y)



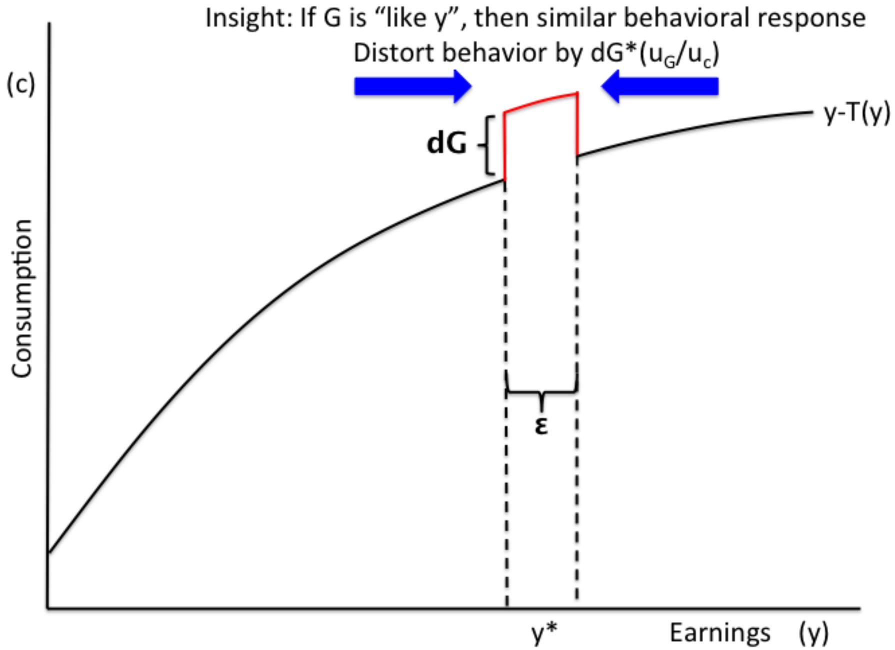
(c)
Increase G to those earning near y*
dG
y-T(y)
Consumption
ε
y*
Earnings (y)

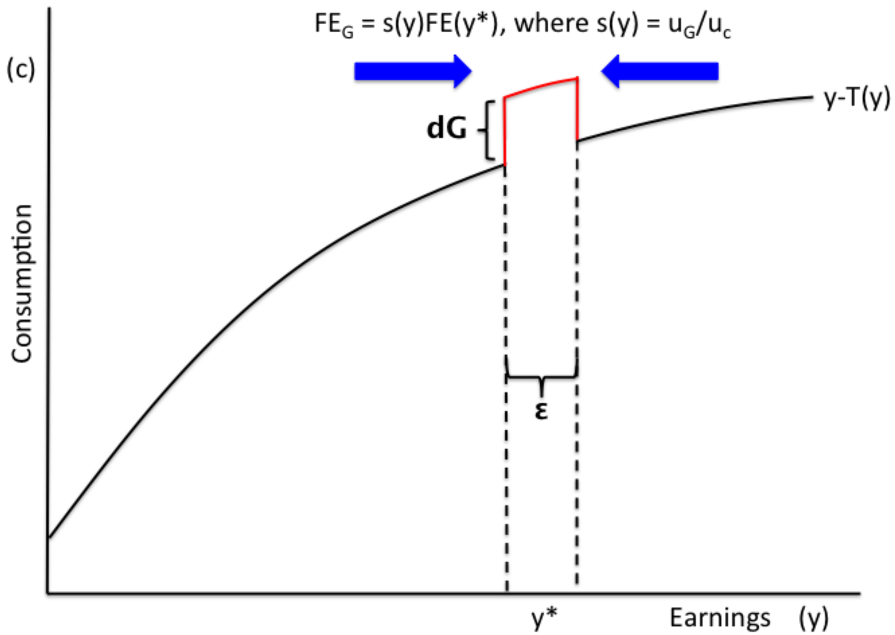


Value: dG*(u_G/u_c)
(c)
y-T(y)
dG
Consumption
ε
y*
Earnings (y)



How big are behavioral responses?
(c)
Consumption
y-T(y)
dG
ε
y*
Earnings (y)


Insight: If G is "like y", then similar behavioral response
Distort behavior by dG*(u_G/u_c)
(c)
Consumption
y-T(y)
dG
ε
y*
Earnings (y)



FE_G = s(y)FE(y*), where s(y) = u_G/u_c
(c)
dG
y-T(y)
Consumption
ε
y*
Earnings (y)

Basic Idea

- If "G is like y", then $\int s(y) FE(y) dy = FE_G$, so that additional G can generate a potential Pareto improvement iff aggregate (unweighted!) surplus is positive:

$$\int s(y)\, dy > 0$$

- Key question: What does it mean for G to be "like y"?
  - Will formalize as weak separability of utility

This Lecture

- Explore these ideas in two broad context that have been focus of previous literature
  - Public goods: Do we subsidize if public good disproportionately helps poor?
    - Follow Kaplow (2006) European Economic Review, "Public Goods and the Distribution of Income"
  - Commodities / in-kind subsidies: Do we subsidize if commodity disproportionately consumed by poor?
    - Follow Kaplow (2006) Journal of Public Economics
- Along the way, discuss other implications/related results
  - Diamond-Mirrlees "Production efficiency" result
  - Zero capital taxation result

Public Goods: Background (Samuelson 1954)

- What are Pure public goods?
  - Non-rival: My consumption doesn't prevent your consumption
  - Non-excludable: Provider can't prevent consumption by those who don't pay
- Public Goods benefit several individuals simultaneously
  - Lowers effective cost of additional G
- Why might the free market under-provide public goods?
  - Free-riding
  - Public goods create positive externalities, individuals under-provide

Optimal Public Goods (Samuelson 1954)

- First Welfare Theorem: Any market equilibrium is Pareto Optimal
  - With public goods, this fails
  - Samuelson (1954) derives condition for a Pareto Optimum
- Consider First Welfare Theorem setup:
  - Individuals indexed by i, two goods, X and G
  - Utility functions $U^i(x_i, G_i)$, standard budget constraint
  - c is the dollar cost of producing G. (Normalize price of x to 1 so $\frac{p_G}{p_x} = c$)
- Condition for private optimality
  - $s_i = \frac{U_G(x_i, G_i)}{U_x(x_i, G_i)} = c \iff s_i = c \ \forall i$

Optimal Public Goods: Failure of FWT

- Now, suppose G is a public good
  - So each person purchases G_i, but values $G = \sum_i G_i$
  - Utility is $U(x_i, G) = U(x_i, G_i + \sum_{j \neq i} G_j)$
- Condition for private optimality
  - Still $\frac{U_G(x_i,G)}{U_x(x_i,G)} = c \iff s_i = c \; \forall i$
    - FOC will determine private contribution to public good
- But, unweighted social surplus is maximized when

$$\sum_i s_i = c$$

Solution: Govt Provision

- Can the government help?
  - Direct provision can avoid the free-rider problem
- What is the optimal level of public provision of G?
  - Samuelson (1954): Pareto efficiency requires maximizing surplus:

$$\sum_i s_i = c$$

- How can we decentralize this?
  - If $\sum MRS_i = c$, then government can find transfers, t_i, and a change in g to make everyone better off
    - Set $t_i = MRS_i$
  - But, if we have individual specific lump-sum transfers, what does this say about the social marginal utility of income for rich and poor?
    - Should be equalized!

Optimal Public Goods

- But, we transfer based on observed income
  - Implies transfers are distortionary!
- What does this mean for optimal public goods? Can still consider taxing back the benefits to each individual i:

$$\int s_i \left(1 + FE\left(y_i\right)\right) di \geq^{?} 1 + FE_G$$

But, now we need to estimate $FE\left(y\right)$ and FE_G!

  - Can we do something simpler?

Kaplow (2006, Euro Econ Review)

- Utility is a function of:
  - A (private) consumption good, c
  - The level of government expenditure on a publicly provided good, g (same as "G" in previous lectures)
  - Labor supply l
- Utility satisfies weak separability: there exists a function v (common to all individuals) such that utility is given by

$$u\left(v\left(c,g\right),l\right)$$

- Individuals differ in their wage, w
- Consumption given by budget constraint

$$c = wl - T\left(wl,g\right)$$

where $T\left(wl,g\right)$ is the tax/transfers to individuals with earnings wl
  - Cannot transfer based on (unobserved) wage, w

Kaplow (2006, Euro Econ Review)

- Social welfare given by

$$SW = \int W\left(U\left(v\left(c, g\right), l\right)\right) f\left(w\right) dw$$

- Government revenue given by

$$R = \int T\left(wl\left(w\right), g\right) f\left(w\right) dw$$

where $l\left(w\right)$ is the labor supply choice of type w

- Social objective: Choose g to maximize SW subject to $R = g$

Kaplow (2006): Benefit Absorbing Tax

- What is the optimal level of g?
- Consider a policy that increases g by a small amount
- Define a "benefit-absorbing tax" (analogous to last lecture...)
    - Change T such that utility does not change when both g and T are simultaneously changed
    - Assume for now that l will not change (will verify later)
    - Will solve implicitly for what the change in the tax schedule must be
- The total derivative from the policy is given by:

$$\frac{\partial U}{\partial g} = \frac{\partial U}{\partial v}\left[v_c c_g + v_g\right]$$

    - $v_c = \frac{\partial v}{\partial c}$ and $v_g = \frac{\partial v}{\partial g}$
    - $c_g = -\frac{\partial T(wl,g)}{\partial g}$ is the partial derivative of how much consumption changes in response to the policy that simultaneously increases g and changes taxes so that utility is unchanged
- We assume that the change in g and increase in T is defined such that $\frac{\partial U}{\partial g} = 0$

Kaplow (2006): Benefit Absorbing Tax

- What must the tax adjustment look like to set $\frac{\partial U}{\partial g} = 0$?
  - i.e. how do we change T in response to the increase in g to hold utility constant for everyone?
- For each level of labor earnings, wl, define the marginal change in the tax schedule by
$$\frac{\partial T(wl, g)}{\partial g} = \frac{v_g}{v_c}$$
Note that this is the individual's WTP for g in units of g.
  - We "tax back the benefits"
- Notice that if we substitute $\frac{\partial T(wl,g)}{\partial g} = \frac{v_g}{v_c}$ into $\frac{\partial U}{\partial g} = \frac{\partial U}{\partial v}\left[v_c c_g + v_g\right]$ we obtain $\frac{\partial U}{\partial g} = 0$ for each type w!

Kaplow (2006): Benefit Absorbing Tax

- But, does the benefit-absorbing tax affect labor supply choices, l?
  - We assumed these were constant...need to verify.
    - This is where weak separability helps
- Define $v(l) = v(wl - T(wl, g), g)$ to be the level of $v(c, g)$ experienced by type w if she chooses l
  - Labor supply l maximizes

$$l(w) = argmax\ U(v(l), l)$$

- Kaplow: Notice that when the policy changes, $v(l)$ is unaffected by the policy change!

$$\frac{dv}{dg}(l) = v_c \frac{\partial T}{\partial g} + v_g = 0 \quad \forall w$$

- Therefore solution to *argmax* $U(v(l), l)$ is not affected by the policy change
  - Graphically: Blue arrows for tax adjustment perfectly offset blue arrows from change in g
  - Exercise: Verify this by solving for $l(w, g)$ and showing that $\frac{\partial l}{\partial g} = 0$ for all w in this policy change.

Kaplow (2006): Aggregate Surplus

- What is the optimal level of public expenditure on g?
- Dual: Maximize government revenue subject to utility held constant

$$\frac{dR}{dg} = \underbrace{\int \frac{dT\left(wl, g\right)}{dg} f\left(w\right) dw}_{\text{Revenue from Benefit-Tax}}$$

- But, note that $\frac{dT(wl,g)}{dg} = \frac{v_g}{v_c} = \frac{U_v}{U_v}\frac{v_g}{v_c} = \frac{\frac{dU}{dg}}{\frac{dU}{dc}} = s\left(y\right)$ is each type's willingness to pay $(y = wl)$
- Re-writing in notation from last class, optimal to increase g whenever aggregate (unweighted!) surplus is positive

$$\int s\left(y\right) dy \geq 1$$

Role of Weak Separability

- What is the role of weak separability? $U(c, g, l) = U(v(c, g), l)$?
- Ensures behavioral response to g is similar to behavioral response for tax cut:

$$FE_G = \int s(y) FE(y) dy$$

- Why might weak separability be violated?
- Suppose g is:
  - Job training
  - Medical care
  - Education
  - Food stamps

Commodity Taxation

- What about commodity taxes? Or taxes on other goods?
  - Subsidize food vs. expensive cars?
- Key papers: Atkinson and Stiglitz (1976) JPubEc and Hylland and Zeckhauser (1979, Scandinavian Journal of Economics)
  - Follow Kaplow (2006, JPubEc) for a nice proof

Kaplow (2006)

- Setup: individuals indexed by h
- Individuals choose commodities $\{c_1, c_2 ...\}$ and labor effort, l
- Maximize utility function

$$u_h\left(c_1, c_2, ..., l\right) = \tilde{u}_h\left(v\left(c_1, ...\right), l\right)$$

  - **Key assumption:** g is the same across people (but $\tilde{u}_h$ can be heterogeneous)
- Subject to budget constraint

$$\sum \left(p_i + \tau_i\right) c_i \leq wl - T\left(wl\right)$$

where w is an individual's wage (heterogeneous in population)
- wl is earnings and $T\left(wl\right)$ is the (nonlinear) tax on earnings

Statement

- Suppose there is a commodity tax

$$\frac{p_i + \tau_i}{p_j + \tau_j} \neq \frac{p_i}{p_j}$$

  for some i and j

- Can welfare be improved by re-setting $\tau_i = \tau_j = 0$ and suitably augmenting the tax schedule T?
  - Atkinson-Stiglitz/Kaplow: YES.
- Define $V(\tau, T, wl)$ to be

$$V(\tau, T, wl) = max\ v(c_1, c_2, ...)$$

$$s.t. \sum (p_i + \tau_i) c_i \leq wl - T(wl)$$

- V is the value of the consumption argument of the utility function – holds independent of labor effort l!
  - Consumption allocations don't reveal any information about labor supply type w conditional on wl.

Proof

- Define intermediate environment:
  - Start with commodity taxes τ
  - Define new taxes at zero $\tau_i^* = 0$
  - Augment the tax schedule
    - Define T^* to offset the impact on utility so that utility is held constant in this intermediate world
  - Specifically, T^* satisfies

$$V(\tau, T, wl) = V(\tau^*, T^*, wl)$$

for all wl

Proof (Cont'd)

- Lemma 1: Every type w chooses the same level of labor effort under τ^*, T^* as under τ, T.
- Proof:
  - Note that

$$U(\tau, T, w, l) = u(V(\tau, T, wl), l) = u(V(\tau^*, T^*, wl), l) = U(\tau^*, T^*, w, l)$$

  - The utility **function** (as a function of l) is the same in both environments
  - Therefore, the l that maximizes utility in the original world maximizes utility in the intermediate world

Proof Cont'd

- Lemma 2: The augmented world raises more revenue than the original world
- Proof:
  - Will show that no individual in the intermediate regime can afford the original consumption vector
    - Implies they pay more taxes in intermediate regime
- Suppose type w can afford original vector when there is no commodity tax, $\tau_i^* = 0$.
  - Then she strictly prefers a different vector because of change in relative price
    - Utility level hasn't changed, but relative prices have
  - But this would imply intermediate environment is strictly better off
    - Choosing a better bundle than the old bundle would strictly increase utility
  - Contradicts definition of intermediate environment holding utilities constant
    - Therefore, type w cannot afford the original bundle

Proof Cont'd

- Next: If type w cannot afford original bundle, then aggregate tax revenue must be higher in the intermediate environment
- Because the original bundle is unaffordable, we have:

$$\sum (p_i)\, c_i > wl - T^*(wl)$$

  for all wl (note $\tau^* = 0$)
- Budget constraint in initial regime implies

$$\sum_i (p_i + \tau_i)\, c_i = wl - T(wl)$$

  - so that

$$\sum_i p_i c_i = -\sum_i \tau_i c_i + wl - T(wl)$$

  - So that

$$-\sum_i \tau_i c_i + wl - T(wl) > wl - T^*(wl)$$

  - or

$$T^*(wl) > \sum \tau_i c_i + T(wl)$$

Proof Cont'd

- So, the intermediate world generates more tax revenue and holds utility constant
- Why does this mean one can have a Pareto improvement from no commodity tax?
- Generate a third world that gives ϵ benefits to everyone through lowering the tax schedule
  - Implies everyone better off.

Implications of Atkinson Stiglitz

- Result generally known as the “Atkinson-Stiglitz” theorem
  - Arguably first shown by Hylland and Zeckhauser (1979)
- Incredibly powerful theorem
- Nests many other results:
  - Zero capital taxes in the standard model
  - “Production efficiency” theorem of Diamond and Mirrlees (1971)

Capital Taxes

- Should we have a tax on capital?
  - Capital owners are rich, doesn't this mean we should tax them if we have redistributive preferences?
- Suppose

$$U(c_1, c_2, ..., l) = u(c_1) - v(l_1) + \beta [u(c_2) - v(l_2)] + ...$$

- With budget constraint

$$\sum_i (p_i + \tau_i) c_i \leq \sum_i w_i l_i$$

- So

$$g(c_1, c_2, ...) = u(c_1) + \beta u(c_2) + ...$$

  - Implies no distortion in relative price of c_1 and c_2
    - You should prove extension to case with l_i instead of just l.
  - What if more productive types have higher preferences for bequests?

Production Efficiency

- Should we let firms deduct the price of inputs
  - E.g. firms don't pay sales tax on their inputs?
- Diamond and Mirrlees (1971) show a surprising result:
- Suppose C is produced with a bunch of intermediate inputs, x_i

$$C = f\left(x_1, ..., x_n\right)$$

  - Question: would you ever want to tax these inputs?
- Answer: No if C is all people care about

$$u\left(x, l\right) = U\left(C\left(x\right), l\right)$$

- The production function for C is the same for all people
  - Weak separability holds
  - Implies no taxes on intermediate inputs

When does weak separability fail?

- When does this fail?
    - Is labor supply an “intermediate input”
        - No taxes on earnings!?
    - What if we can’t tax profits of an intermediate producer?

Relation to Mirrlees

- Another way of seeing this: Mirrlees information logic:
  - When commodity choices have desirable information about type conditional on earnings?
    - See Mirrlees (1976, JPubEc)
- What constitutes "desirable information"? (Saez 2002 JPubEc)
  - Information about social welfare weights: Society likes people that consume x_1 more than x_2 conditional on earnings
    - Implement subsidy on good x_1 financed by tax on x_2
    - First order welfare gain (b/c of difference in social welfare weights)
    - Second order distortionary cost starting at $\tau = 0$
  - Information about latent productivity: More productive types like x_1 more than x_2 conditional on earnings
    - e.g. x_1 is books; x_2 is surf boards
    - Then, tax the goods rich people like but reduce the marginal tax rate
    - Leads to increase in earnings!
    - Depends on covariance

Key Lessons

- In general, need to estimate fiscal externalities associated with policy changes
- But, if willing to assume weak separability of utility, can just assume that the FE is the same as an income tax
- Motivates only needing to calculate whether the aggregate surplus is positive
  - Are people WTP for the policy change out of their own income?

Two empirical literatures on Public Goods

- Two empirical literatures on public goods:
  - Measuring willingness to pay
  - Measuring private crowd-out of government provision

Measuring WTP

- Two methods:
  - Infer based on behavior / prices
  - Ask people (Contingent valuation)

Value of Clean Air

- How would you measure the WTP for clean air?
- Brookshire et al. (1982)
  - Infer willingness to pay for clean air using effect of pollution on property prices (capitalization)
- Let P_i denote house price of house i, regress

$$P_i = \alpha + \beta Pollution_i + \gamma X_i + \epsilon_i$$

  for range of controls, X_i.
- Concerns?

Value of Clean Water

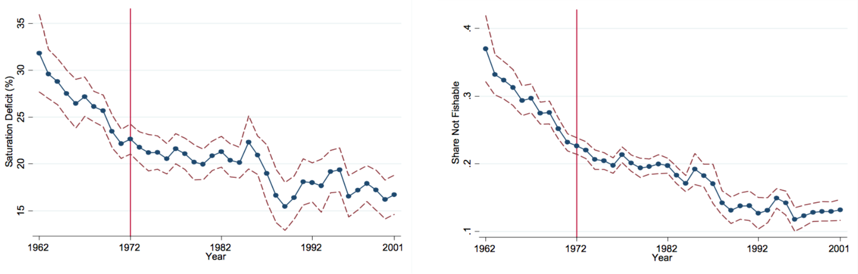
- More recently, Keiser and Shapiro (2017): "Consequences of the Clean Water Act and the Demand for Water Quality"
  - Cost-benefit analysis of the Clean Water Act
- Three analyses
  - Estimate water pollution from 1962-2001
  - Estimate impact of clean water act grants to wastewater treatment plants on pollution
  - Estimate WTP for clean water grants from house prices within 25 mi of plants

Keiser and Shapiro (2017)

Figure 2. Water Pollution Trends, 1962-2001

Panel A. Dissolved Oxygen Deficit

Panel B. Share Not Fishable



Notes: Graphs show year fixed effects plus a constant from regressions which also control for monitoring site fixed effects, a day-of-year cubic polynomial, and an hour-of-day cubic polynomial, corresponding to equation (2) from the text. Connected dots show yearly values, dashed lines show 95% confidence interval, and 1962 is reference category. Standard errors are clustered by watershed.

Keiser and Shapiro (2017)

$$Q_{pdy} = \sum_{\tau=-10}^{\tau=25} \gamma_\tau 1[G_{p,y+\tau} = 1] d_d + X'_{pdy}\beta + \eta_{pd} + \eta_{py} + \eta_{dwy} + \epsilon_{pdy}$$

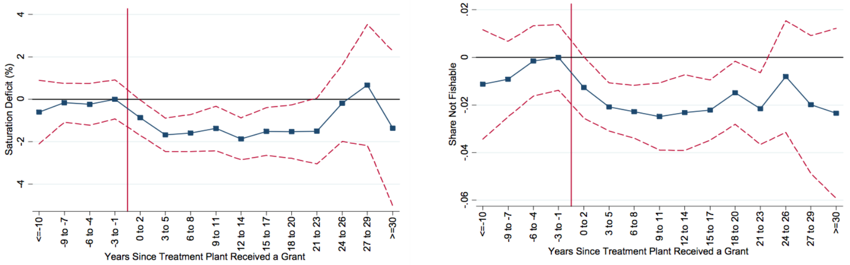
- Event-study design:
  - Two observations for each treatment plant: one upstream and one downstream
    - $G_{p,y+\tau}$ indicator for grant received in year $y+\tau$, where τ indexes years since grant received
    - d_d is an indicator for being downstream from the treatment facility
    - X_{pdy} are controls for temperature and precipitation
    - plant-downstream fixed effects, η_{pd} allow for different mean levels up and down-stream
    - plant-year fixed effects, η_{py}, control for forces like growth of local industry/etc that affect water quality
    - downstream-by-basin-by-year, η_{dwy}, allow upstream and downstream water quality to differ by year in ways common to all plants in a river basin

Keiser and Shapiro (2017)

Figure 3. Effects of Clean Water Act Grants on Water Pollution: Event Study Graphs

Panel A. Dissolved Oxygen Deficit

Panel B. Share Not Fishable



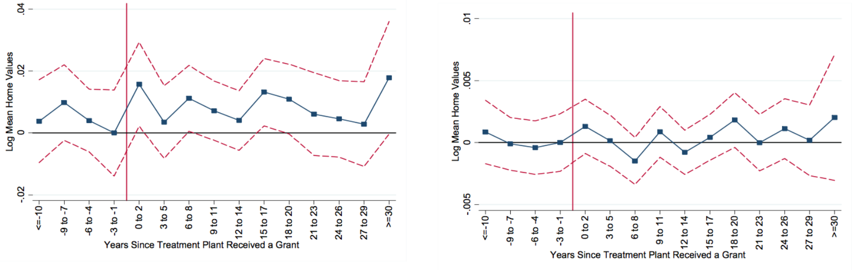
Notes: Graphs show coefficients on downstream times year-since-grant indicators from regressions which correspond to the specification of Table 3. These regressions are described in equation (5) from the main text. Data cover years 1962-2001. Connected dots show yearly values, dashed lines show 95% confidence interval. Standard errors are clustered by watershed.

Keiser and Shapiro (2017)

Figure 4. Effects of Clean Water Act Grants on Log Mean Home Values: Event Study Graphs

Panel A. Homes Within 0.25 Miles of River

Panel B. Homes Within 25 Miles of River



Notes: Graphs show coefficients on year-since-grant indicators from regressions corresponding to the specification of Table 6, column (3). Connected dots show yearly values, dashed lines show 95% confidence interval. Standard errors are clustered by watershed. Panels A and B show different ranges of values on their y-axes. Data cover decennial census years 1970-2000.

Value of Clean Water

- Conclusion: Impact on house prices in 25 mile radius is $< 1/3$ of the costs
- Concerns?
- Distributional incidence?

Optimal Taxation in Ramsey (1927)

- Ramsey (1927): How should commodities be taxed to raise revenue, $R > 0$.
    - Modeled by Diamond and Mirrlees (1971)
- Key result: Tax-weighted Hicksian price derivatives are equated across goods
    - "Inverse elasticity rule": tax goods with smaller compensated behavioral responses

Setup

- Representative Agent (drop i subscripts).
- Commodities, x_k, indexed by k
- Government imposes taxes on commodities, τ_k.
- Necessary condition for optimality

$$\frac{d\hat{V}_P}{d\theta}|_{\theta=0} = 0$$

  for all feasible policy paths P.
- Optimal tax would be lump-sum of size R
  - Assumed to not exist

Commodity Tax Variation

- Consider policy $P(\theta)$ that changes commodity taxes (e.g. lowers tax on good 1 and raises tax on good 2)
- Budget neutral: $\frac{d\hat{t}}{d\theta} = 0$
- No change in public goods
- So, optimality condition only involves behavioral response:

$$\sum_k \hat{\tau}_k \frac{d\hat{x}_k}{d\theta}|_{\theta=0} = 0$$

Hicksian Elasticity

- Diamond and Mirrlees (1971): At the optimum, expand the behavioral response using the Hicksian demands, x_k^h,

$$\frac{dx_k}{d\theta} = \frac{\partial x_k^h}{\partial \tau_1}\frac{d\tau_1}{d\theta} + \frac{\partial x_k^h}{\partial \tau_2}\frac{d\tau_2}{d\theta}$$

  - Additional term, $\frac{\partial x_k^h}{\partial u}\frac{dV_p}{d\theta}$, but this vanishes at the optimum.
- Optimality condition is given by

$$\sum_k \tau_k \frac{\partial x_k^h}{\partial \tau_1}\frac{d\tau_1}{d\theta} = \sum_k \tau_k \frac{\partial x_k^h}{\partial \tau_2}\left(-\frac{d\tau_2}{d\theta}\right)$$

  - Tax-weighted Hicksian responses are equated across the tax rates
    - Inverse elasticity rule
- What are the needed elasticities?

Inverse Elasticity Rule

- Assume cross elasticities are zero:

$$BC = x_1 \frac{d\tau_1}{d\theta} + \tau_1 \frac{dx_1}{d\theta} + x_2 \frac{d\tau_2}{d\theta} + \tau_2 \frac{dx_2}{d\theta} = 0$$

  so

$$x_1 \left(1 + \frac{\tau_1}{x_1} \frac{\partial x_1^h}{\partial \tau_1}\right) \frac{d\tau_1}{d\theta} = x_2 \left(1 + \frac{\tau_2}{x_2} \frac{\partial x_2^h}{\partial \tau_2}\right) \left(-\frac{d\tau_2}{d\theta}\right)$$

- And optimality implies

$$x_1 \left(\frac{\tau_1}{x_1} \frac{\partial x_1^h}{\partial \tau_1}\right) \frac{d\tau_1}{d\theta} = x_2 \left(\frac{\tau_2}{x_2} \frac{\partial x_2^h}{\partial \tau_2}\right) \left(-\frac{d\tau_2}{d\theta}\right)$$

Inverse Elasticity Rule

- So

$$\left(\frac{\tau_1}{x_1}\frac{\partial x_1^h}{\partial \tau_1}\right) = \left(\frac{\tau_2}{x_2}\frac{\partial x_2^h}{\partial \tau_2}\right) = \kappa$$

- Translating to price (1+tau) instead of tax (tau) elasticities:

$$\frac{\tau_j}{1+\tau_j}\epsilon^h_{j,(1+\tau_j)} = \kappa$$

Or

$$\frac{\tau_j}{1+\tau_j} = \frac{\kappa}{\epsilon^h_{j,(1+\tau_j)}}$$

which is the "inverse elasticity rule".

Key Result: Inverse Elasticity Rule

- Main result of Ramsey model: Inverse elasticity rule
- Key Assumptions:
  - Representative agent
  - No lump sum taxation

Optimal Taxation of Production

- Diamond and Mirrlees (1971) also consider the issue of production efficiency.
- Commodities, x_k, indexed by k, transformed into one another (produced) by firms and government
- Producer prices p_k, Consumer prices q_k
  - Tax is wedge $\tau_k = q_k - p_k$
- Consumer i solves $\max u_i(\mathbf{x})$ s.t. $\sum q_k x_k \leq 0$
  - Defines consumer (final) demand for each commodity $x_k^i(\mathbf{q})$
  - and indirect utility $V_i(\mathbf{q}) = u(\mathbf{x}^i(\mathbf{q}))$
- Note: Consumers are the ones endowed with the initial commodity supply
- Endowments allow them to exchange, consumers are on budget constraint

Firm side

- Price-taking firms j transform commodities
- Production possibilites represented by input output function $f^j(\mathbf{y}) = 0$
  - for example, $y_1 = y_2^{.3} * y_3^{.7} \iff y_1 - (-y_2^{.3}) * (-y_3^{.7}) = 0$
  - Can turn $y_2$and y_3 into y_1 (or vice versa, depending of domain)
  - Negative arguments are inputs, positives are outputs

Firm side: CRS Production

- Assumption: constant returns to scale
- Then each firm can produce "as much" or "as little" as desired in fixed proportions
  - Together, many CRS firms define an aggregate production function $f(\mathbf{y}) = 0$
  - No profits for any firm (otherwise infinite production) in equilibrium
  - $\mathbf{p} \cdot \mathbf{y}^{\mathbf{j}} = 0$ must hold in equilibrium, and thus $\mathbf{p} \cdot \mathbf{y} = \mathbf{p} \cdot \left(\sum \mathbf{y_j}\right) = 0$
- Under CRS, behavior of many optimizing firms same as one aggregate firm

Firm side: Firm Objective

- Objective: Choose point on frontier to maximize output prices - input prices
- $$\max \mathbf{p} \cdot \mathbf{y} \text{ s.t. } f(y) = 0$$
- Optimality condition: $\frac{\partial f}{\partial y_k} = p_k \iff MRT = \frac{\frac{\partial f}{\partial y_k}}{\frac{\partial f}{\partial y_{k'}}} = \frac{p_k}{p'_k}$
  - Why can we ignore lagrange multiplier on $f(y) = 0$ condition? Because we can normalize the units of f to be in terms of one of the commodities...see Diamond-Mirrlees (1971).

Govt

- D&M think of Gov't as a planner with a distributive objective but:
  - Can't just pick point on PPF
  - Must deal with consumers through market place using uniform prices
  - Uses:
    - a.) linear commodity taxes to set prices and
    - b.) public production to adjust quantities above and beyond what private sector does given prices
- Public production follows PPF given by $g(z) \leq 0$

Objective

- What is the objective here?
  - redistribution–different than Ramsey, since no revenue requirement
- Why would commodity taxes help with no lump sum transfers?
  - differential wealth levels are due to endowment differences
  - Commodity taxes target:
    - Different tastes
    - Value of endowment
  - But commodity taxes cause DWL

Objective

- Solve

$$\max_{q,p,z} \sum_i W(V_i(\mathbf{q})) \text{ s.t. } \sum_i x_k^i(\mathbf{q}) = y_k(\mathbf{p}) + z_k \text{ , } f(\mathbf{y}) = \mathbf{0}, \text{ and } g(\mathbf{z}) = 0$$

- Lagrangian

$$\max_{q,p,z} \sum_i W(V_i(\mathbf{q})) + \sum_k \lambda_k (y_k(\mathbf{p}) + z_k - \sum_i x_k^i(\mathbf{q})) + \gamma^f f(\mathbf{y}(\mathbf{p})) + \gamma^g g(\mathbf{z})$$

Objective

- Production-side and consumer-side variables are additively separable

$$\max_{q,p,z} \underbrace{\sum_i W(V_i(\mathbf{q})) - \sum_k \lambda_k \sum_i x_k^i(\mathbf{q})}_{consumption} + \underbrace{\sum_k \lambda_k (y_k(\mathbf{p}) + z_k) + \gamma^f f(\mathbf{y}(\mathbf{p})) + \gamma^g g(\mathbf{z})}_{production}$$

- Note that FOC for producer prices and government production depend on W only through the shadow value of an endowment unit of k.
- Also, choice of p *directly* implements y, so we can choose y directly

$$\max_{q,y,z} \underbrace{\sum_i W(V_i(\mathbf{q})) - \sum_k \lambda_k \sum_i x_k^i(\mathbf{q})}_{consumption} + \underbrace{\sum_k \lambda_k (y_k + z_k) + \gamma^f f(\mathbf{y}) + \gamma^g g(\mathbf{z})}_{production}$$

Result

- [FOC y_k]$\lambda_k = \gamma^f \frac{\partial f}{\partial y_k}$
- [FOC g_k]$\lambda_k = \gamma^g \frac{\partial g}{\partial z_k}$
- Taking ratio, for any social welfare objective, it must be the case that:

$$\frac{\frac{\partial g}{\partial z_k}}{\frac{\partial g}{\partial z_{k'}}} = \frac{\frac{\partial f}{\partial y_k}}{\frac{\partial f}{\partial y_{k'}}} = \frac{p_k}{p_{k'}}$$

- The government's decision to intervene in the economy is independent of the objective. MRTs are always equalized, and the only wedge is between consumer and producer prices. Production-side and consumer-side variables are additively separable