

Topic 2: Incorporating Redistributive Concerns

Nathaniel Hendren

Harvard

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Distributional Incidence

- Suppose there's a budget-neutral policy that hurts the poor and helps the rich.
- The rich are willing to pay \$1.5 for the policy
- The poor are willing to pay \$0.5 to prevent the policy from going into place
- Should we do the policy?

Distributional Incidence

- Two common economic methods for resolving interpersonal comparisons
- ① Social welfare function (Bergson (1938), Samuelson (1947), Diamond and Mirrlees (1971), Saez and Stantcheva (2015))
 - Allows preference for equity
 - Do the policy only if \$1.50 to the rich is valued more than \$0.5 to the poor:

$$\frac{\eta^{rich}}{\eta^{poor}} > \frac{1}{3}$$

- Subjective choice of researcher or policy-maker
- ② Kaldor Hicks Compensation Principle (Kaldor (1939), Hicks (1939, 1940))
 - Motivates aggregate surplus, or “efficiency”, as normative criteria
 - \$1.50 - \$0.50 = \$1 > 0 \implies do the policy
 - Ignores issues of “equity”

Kaldor Hicks: Motivating Aggregate Surplus

- Suppose individuals, i , are willing to pay s_i for a policy change.
 - Pareto only if $s_i > 0$ for all i
 - In general, $s_i > 0$ and $s_j < 0$ for some i and j
 - What to do?
- Kaldor Hicks: Suppose we consider alternative policy that also has taxes/transfers to individuals, t_i .
 - How much can we tax each individual and break even?
 - Aggregate surplus

$$t_i^{max} = s_i$$

- Potential Pareto improvement if and only if

$$\sum_i t_i^{max} > 0 \iff \sum_i s_i > 0$$

- If total (unweighted) surplus is positive, then the government can institute taxes + the policy to make everyone better off

- Key insight of Kaldor and Hicks: only compare policies that have the same distributional incidence
 - Use individual-specific lump-sum transfers to neutralize interpersonal comparisons
- BUT: Key insight of Mirrlees and optimal tax literature: Can't do individual-specific lump-sum taxes
 - Want to tax two people with the same income differently (high effort low luck vs. low effort high luck)
- This lecture: Modify Kaldor-Hicks so that transfers are incentive compatible (Mirrlees (1971))
 - Apply to MVPF calculations in Topic 1
 - Relate to inverse optimal taxation literature

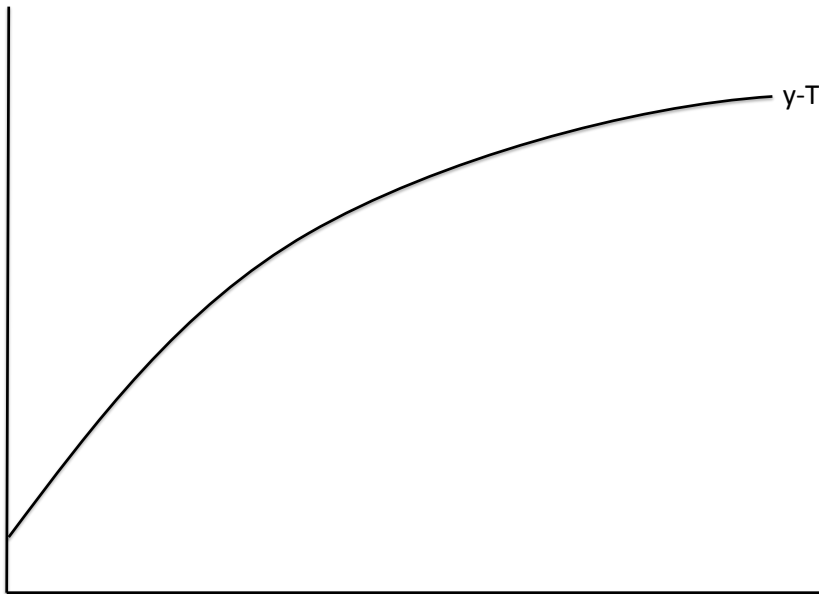
- Hicks (1939) writes:
 - *“If, as will often happen, the best methods of compensation feasible involve some loss in productive efficiency, this loss will have to be taken into account. (Hicks, 1939)”*
- Loosely follow mathematical models in Hendren (2014) “The Inequality Deflator: Interpersonal Comparisons without a Social Welfare Function”
- Other key readings:
 - Main ideas first presented in Mirrlees (1976) (A classic!)
 - Empirically implemented in inverse optimum literature (Bourguignon and Spadaro, 2012)
 - See also Hylland and Zeckhauser (1979), Coate (2000), Kaplow (1996, 2004, 2006, 2008)

Exploiting the Envelope Theorem...

- Key idea: Envelope theorem allows for empirical method to account for distortions
 - Goal: turn unequal surplus into equal surplus using modifications to the tax schedule
 - Not individual-specific lump-sum transfers
 - Cost of moving \$1 of surplus differs from \$1 because of how behavioral response affects government budget
- Suppose we want to provide transfers to those earning near y^*

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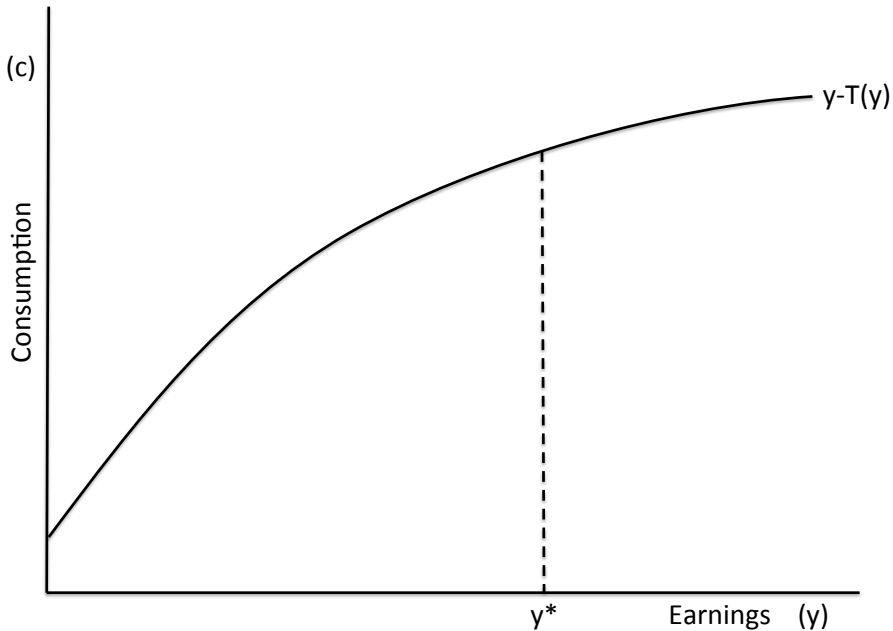
Consumption



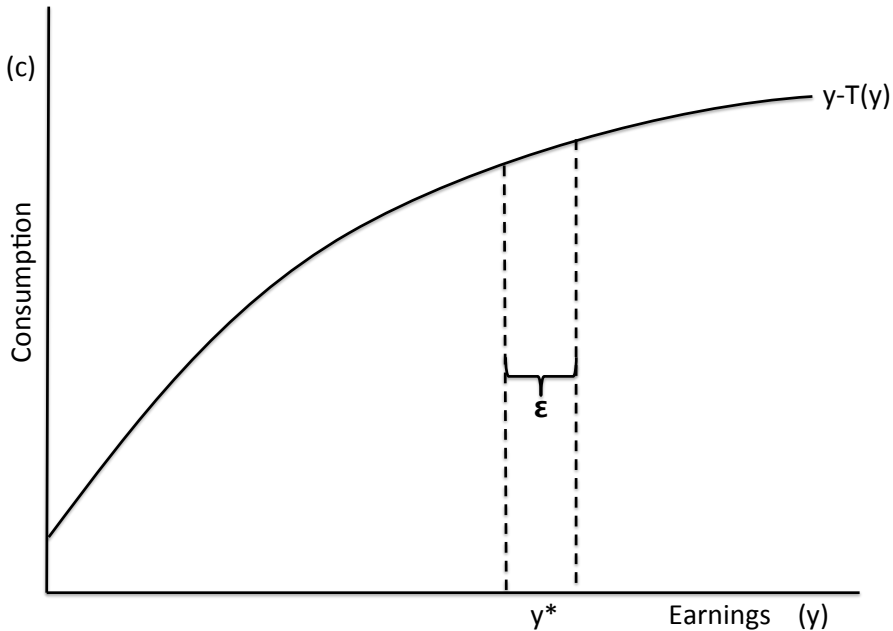
Earnings (y)

$y-T(y)$

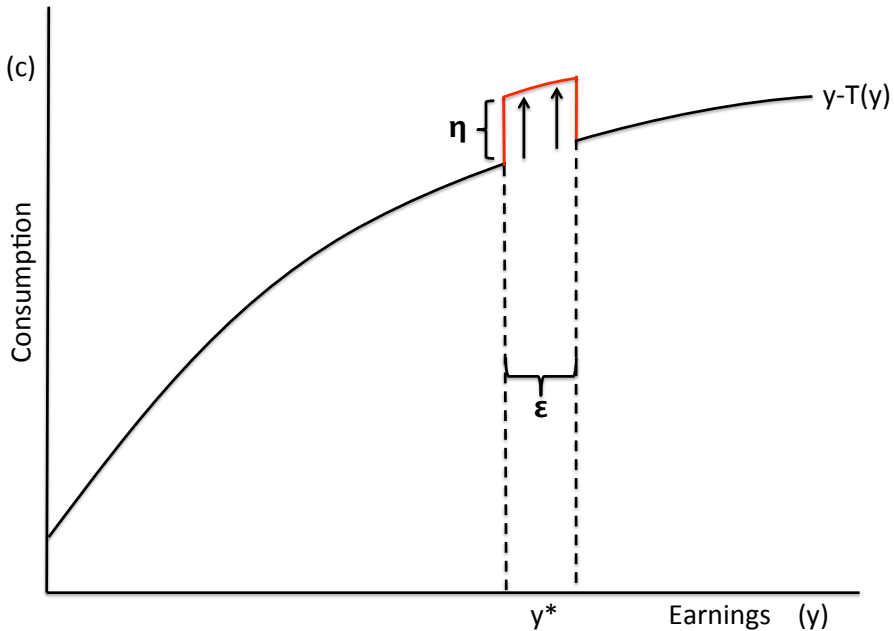
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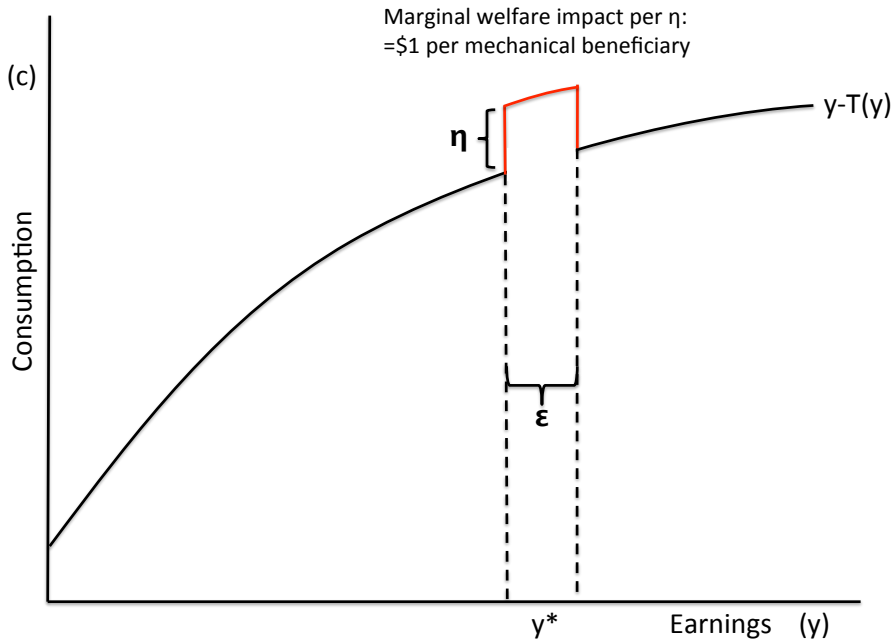


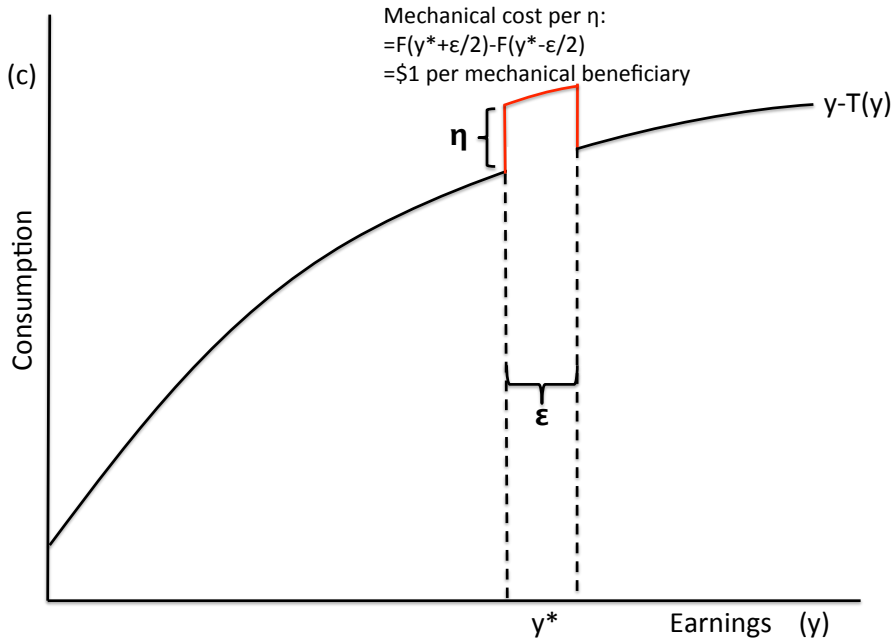
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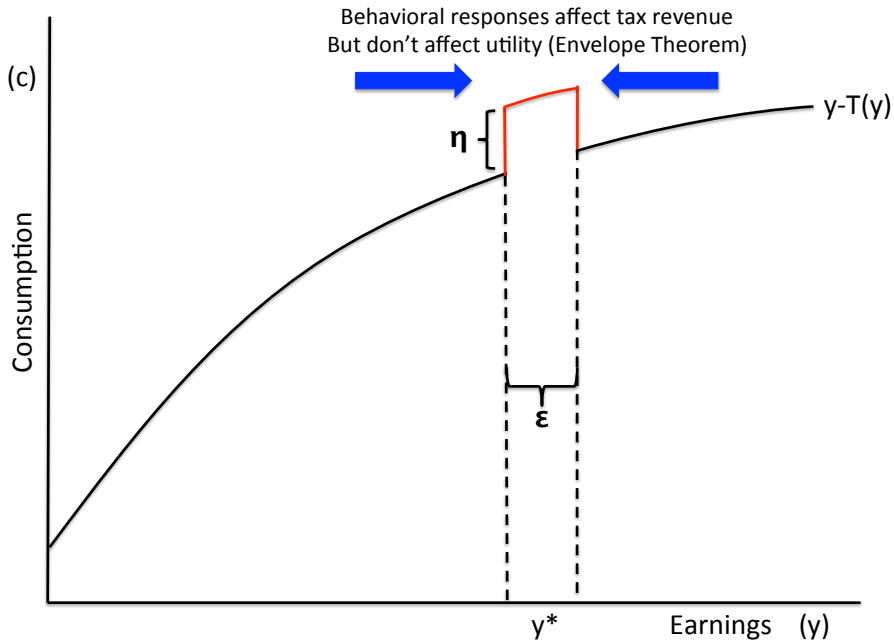


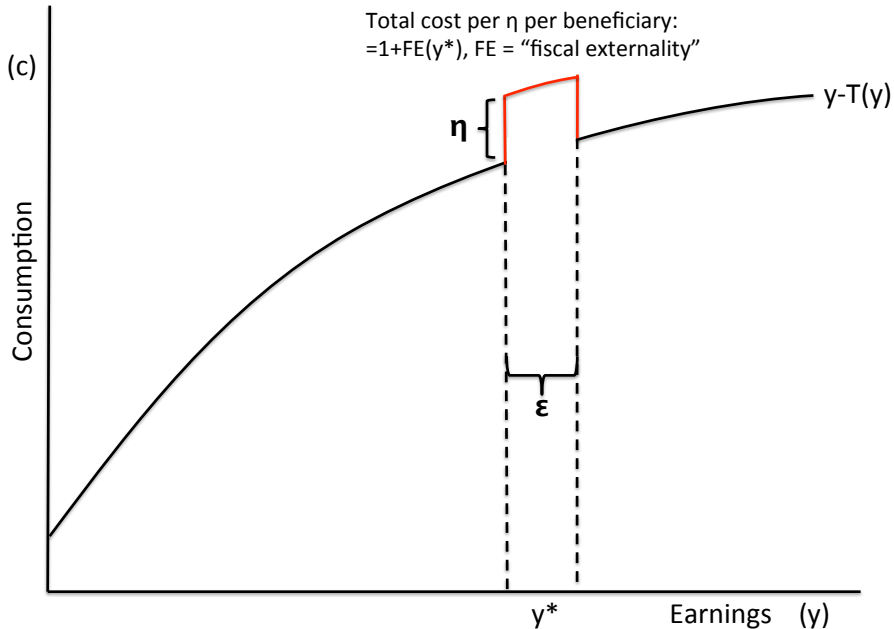
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- Consider the function:

$$g(y) = \frac{1 + FE(y)}{E[1 + FE(y)]}$$

- To first order: \$1 surplus to those earning y can be turned into $\$g(y) / n$ surplus to everyone through modifications to tax schedule
- Fiscal externality logic does not rely on functional form assumptions
 - Allows for each person to have her own utility function and arbitrary behavioral responses
 - Extends to multiple policy dimensions (Later if time...)
- Later: $s(y)$ of surplus to those earning y can be turned into $s(y) \frac{g(y)}{n}$ surplus to everyone
- For now, find empirical expression for $FE(y)$

Mathematical Derivation

- What is the marginal cost of a tax cut to those earning near y ?
- Consider calculus of variations in $T(y)$
 - Define $\hat{T}(y; y^*, \epsilon, \eta)$ by

$$\hat{T}(y; y^*, \epsilon, \eta) = \begin{cases} T(y) & \text{if } y \notin (y^* - \frac{\epsilon}{2}, y^* + \frac{\epsilon}{2}) \\ T(y) - \eta & \text{if } y \in (y^* - \frac{\epsilon}{2}, y^* + \frac{\epsilon}{2}) \end{cases}$$

- \hat{T} provides η additional resources to an ϵ -region of individuals earning between $y^* - \epsilon/2$ and $y^* + \epsilon/2$.
- Given \hat{T} , individual of type θ chooses $\hat{y}(y^*, \epsilon, \eta; \theta)$ that maximizes utility
 - Some people who earn near y^* might move away from y^* because the government is taxing them more (or move towards y^* if $\eta < 0$)

Causal effects (vs. IC constraints)

- Define choice of income, y , in environment with ϵ and η by

$$\hat{y}(\theta; y^*, \epsilon, \eta) = \operatorname{argmax}_y u(y - \hat{T}(y; y^*, \epsilon, \eta), y; \theta)$$

- How does this relate to IC constraints in mechanism design approach?
 - Embedded in \hat{y} function - we substitute the maximization program into the resource constraint and assume observed behavior maximizes the IC constraint
 - Trade causal effects of tax variation for structural assumptions of type distribution and shapes of preferences
 - Causal effects are sufficient...

Marginal Cost of Taxation

- Given choices $\hat{y}(y^*, \epsilon, \eta; \theta)$, government revenue is given by

$$\hat{q}(y^*, \epsilon, \eta) = \frac{1}{\Pr\{y(\theta) \in [y^* - \frac{\epsilon}{2}, y^* + \frac{\epsilon}{2}]\}} \int_{\theta} [\hat{T}(\hat{y}(\theta; y^*, \epsilon, \eta); y^*, \epsilon, \eta) - T(y(\theta))] d\mu(\theta)$$

(normalized by the number of mechanical beneficiaries).

- Note $\hat{q}(y^*, 0, \eta) = \hat{q}(y^*, \epsilon, 0) = 0$ for all ϵ and η
- Marginal cost of a tax cut to those earning near y :

$$1 + FE(y) = \lim_{\epsilon \rightarrow 0} \frac{\partial \hat{q}(y, \epsilon, \eta)}{\partial \eta}$$

- What are the key assumptions to obtain this representation of the cost of taxation?
 - Partial equilibrium / “local incidence”
 - Behavioral response only induces a fiscal externality
 - Other incidence/externalities would need to be accounted for
 - Others?

Two Types of Policies

- Basic Idea: Use $1 + FE(y)$ to weight individual willingness to pay for a policy
 - Implements modified Kaldor-Hicks in which transfers occur through income tax schedule
- Broadly, two types of policies to consider:
 - Changes to the tax schedule
 - Changes to other goods/transfers/etc

Changes to the Tax Schedule

- To begin, what about policies that change the tax schedule?
 - Must be indifferent to these!
 - Why?
 - Suppose the tax schedule goes from $T(y) \rightarrow T(y) + \epsilon h(y)$
 - Let $s_\epsilon(y)$ denote individual y 's WTP for the policy change. And, let $s(y) = \lim_{\epsilon \rightarrow 0} \frac{s_\epsilon(y)}{\epsilon}$ denote the individual's marginal willingness to pay for the tax change
 - Exercise: Show $\int s(y) (1 + FE(y)) = 0$

- But, can we say nothing about welfare of changes to the tax schedule?
- What if $FE(y) < -1$?
 - Impact of behavioral response to tax change is larger than mechanical revenue raised from the tax
 - Local Laffer effect
- Werning 2007 shows that this characterizes when there exists Pareto efficient changes to tax schedule
 - Lowering taxes at y will improve everyone's welfare
 - Those with incomes near y pay less taxes
 - And there's more revenue to the government (which can be redistributed)

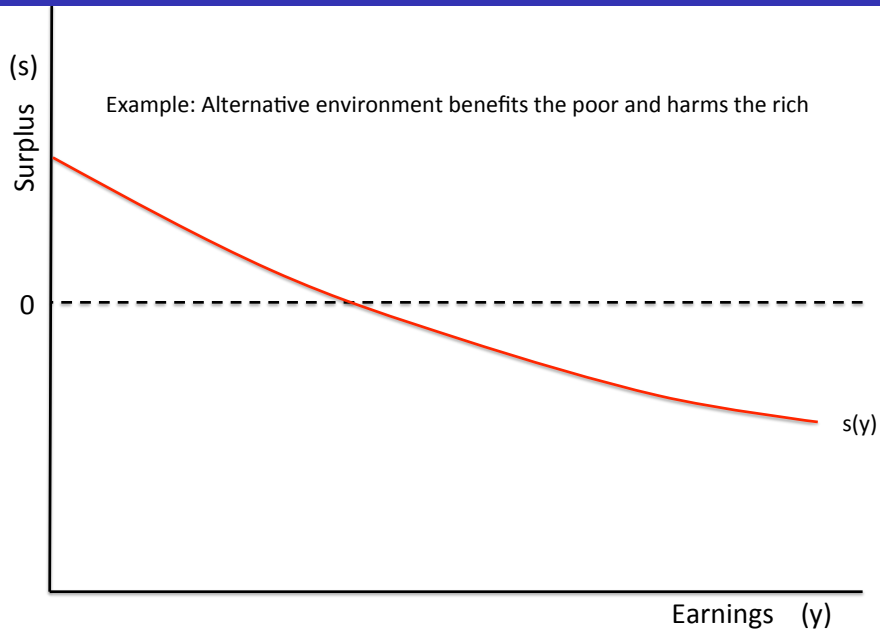
Welfare Analysis of Non-Tax Policies

- What about the welfare impact of other (non-tax) policies?
- Given policy, let $s(y)$ denote the WTP of individual earning y for the policy
 - Assume for simplicity WTP does not vary conditional on y . Given by:

$$s(y) = \frac{\frac{\partial u}{\partial G}}{\lambda}$$

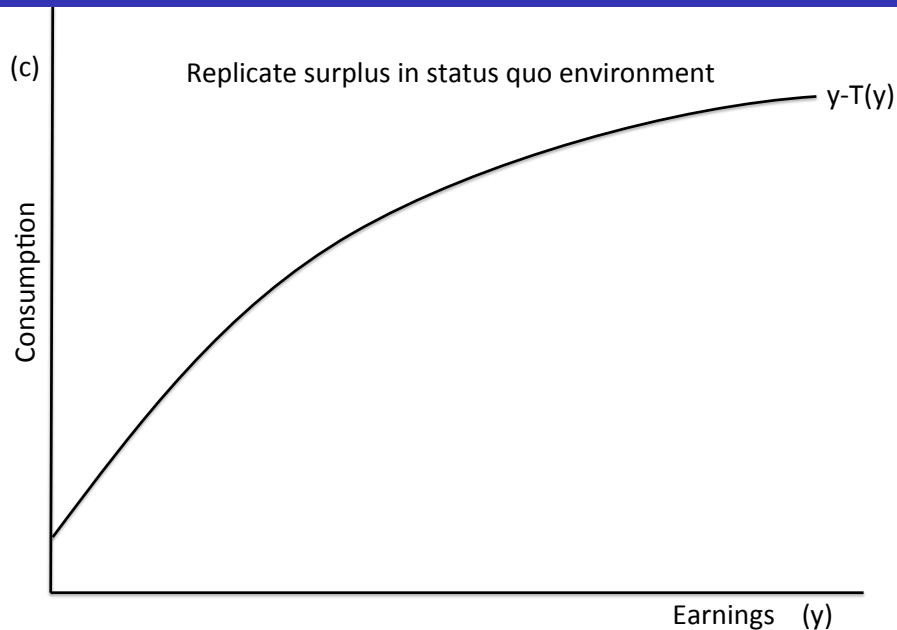
- If $s(y)$ is everywhere positive, then Pareto improvement
- But, how to resolve tradeoffs if $s(y_1) < 0$ and $s(y_2) > 0$?

Example: Alternative Environment Benefits Poor

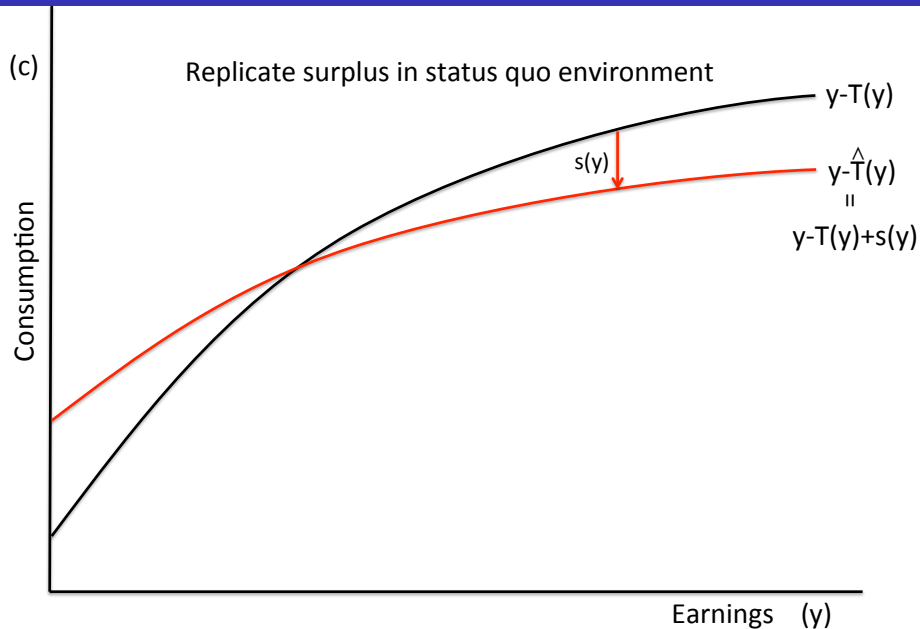


- Given $s(y)$, let's consider a modified policy that neutralizes distributional comparisons
- Two ways of neutralizing distributional comparisons: EV and CV
- “EV”: modify status quo tax schedule
 - By how much can everyone be made better off in modified status quo world relative alternative environment?

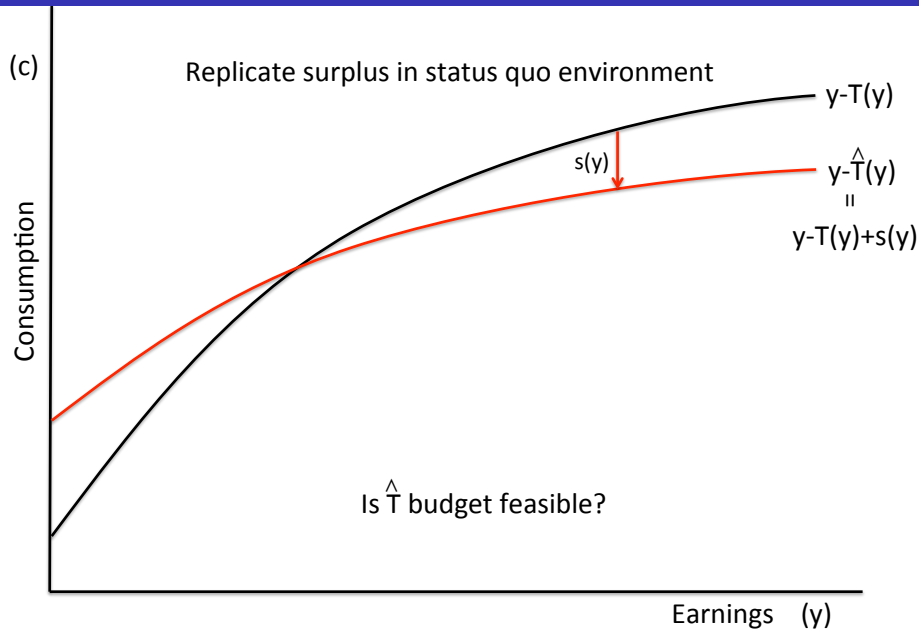
"EV" Example



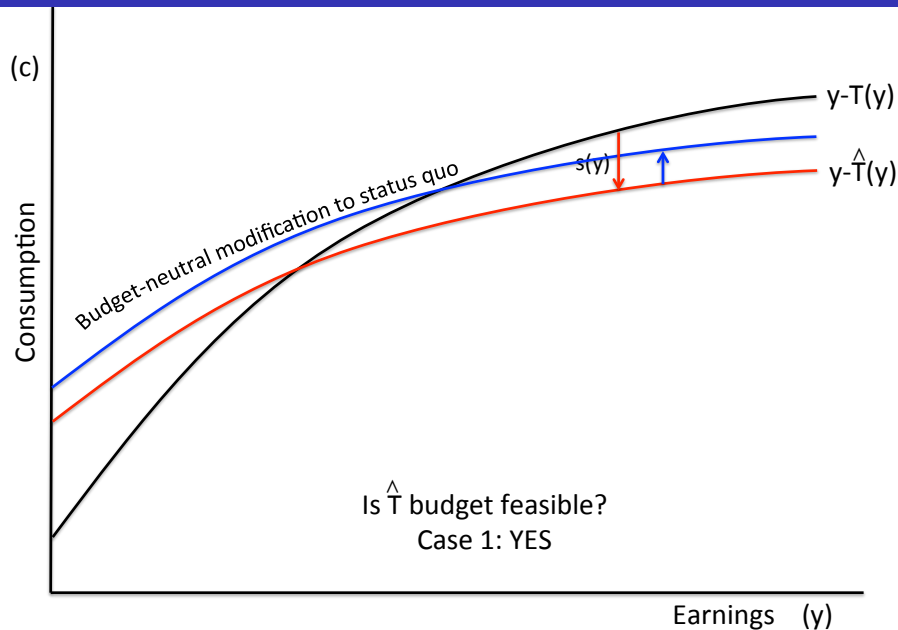
"EV" Example



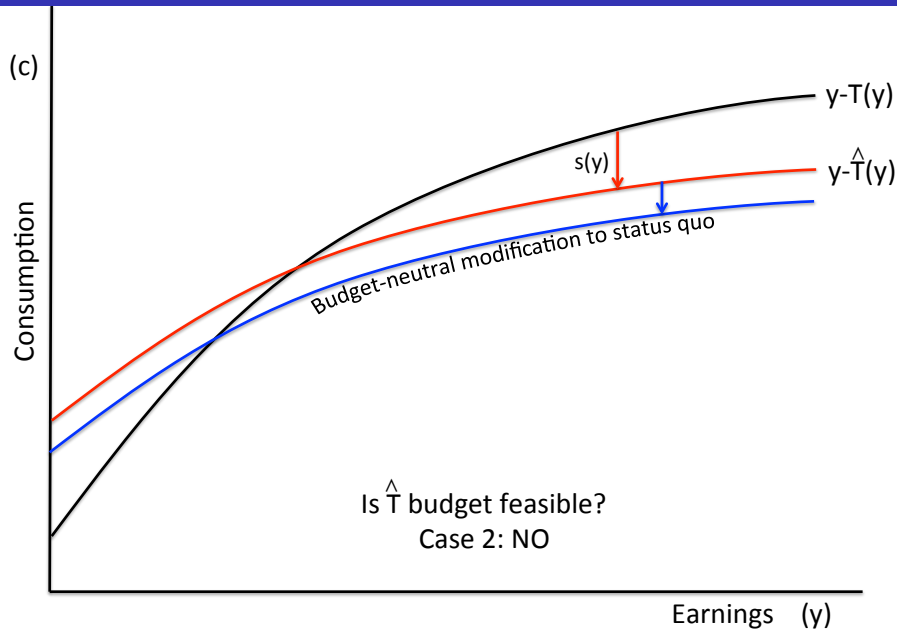
“EV” Example



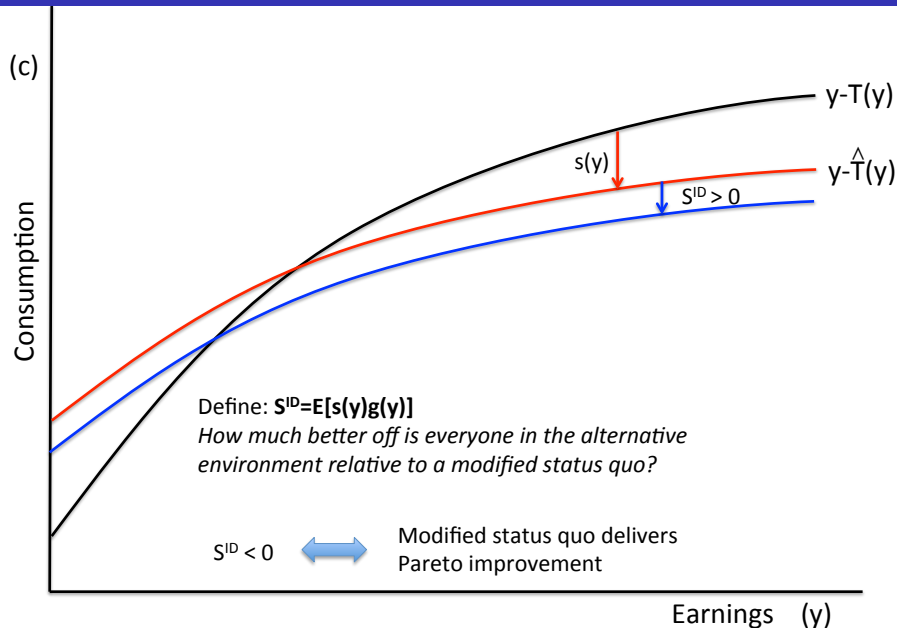
"EV" Example



“EV” Example

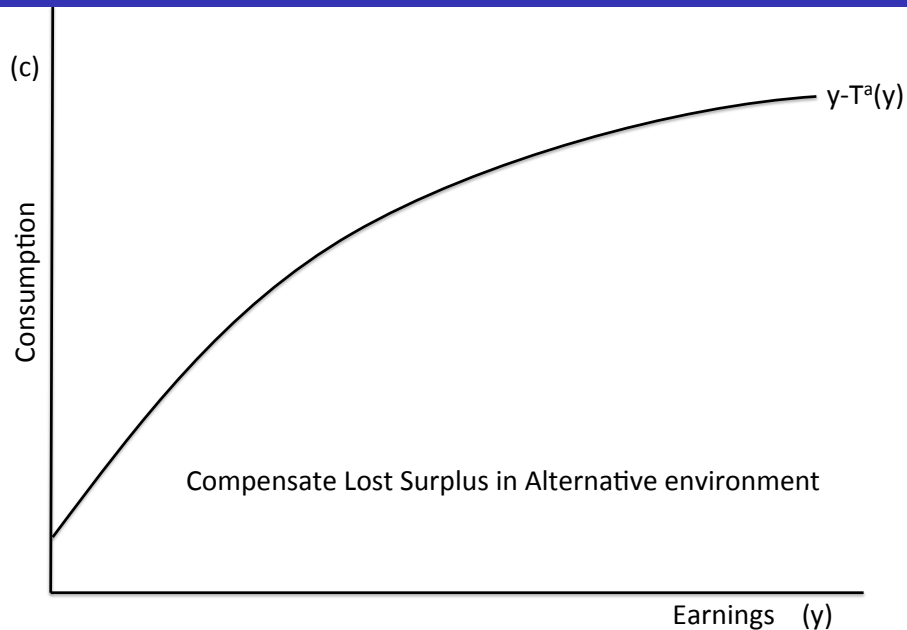


“EV” Example

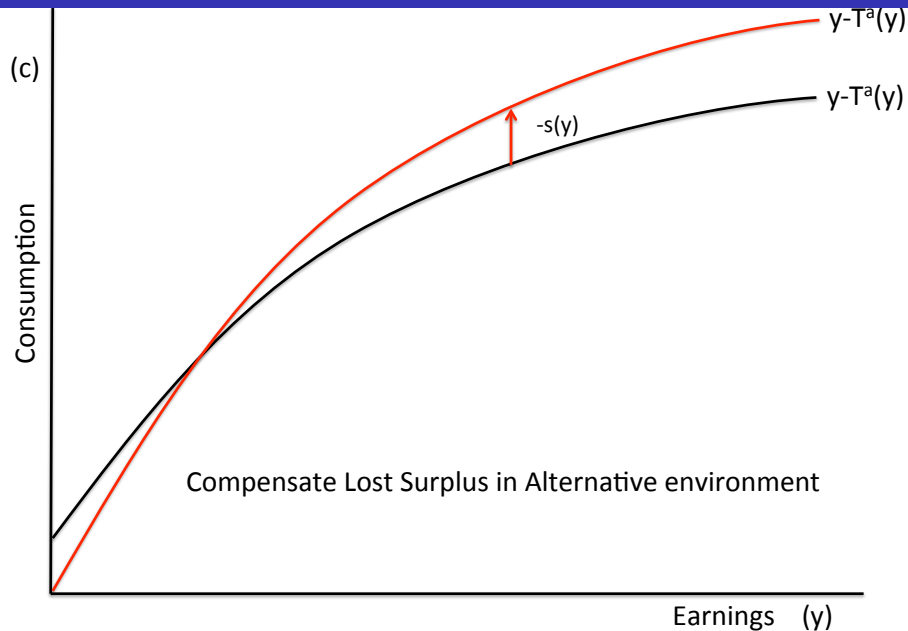


- Given $s(y)$, two ways of neutralizing distributional comparisons
- “EV”: modify status quo tax schedule
 - By how much can everyone be made better off in modified status quo world relative alternative environment?
- “CV”: modify alternative environment tax schedule
 - By how much can everyone be made better off in modified alternative environment relative to status quo?

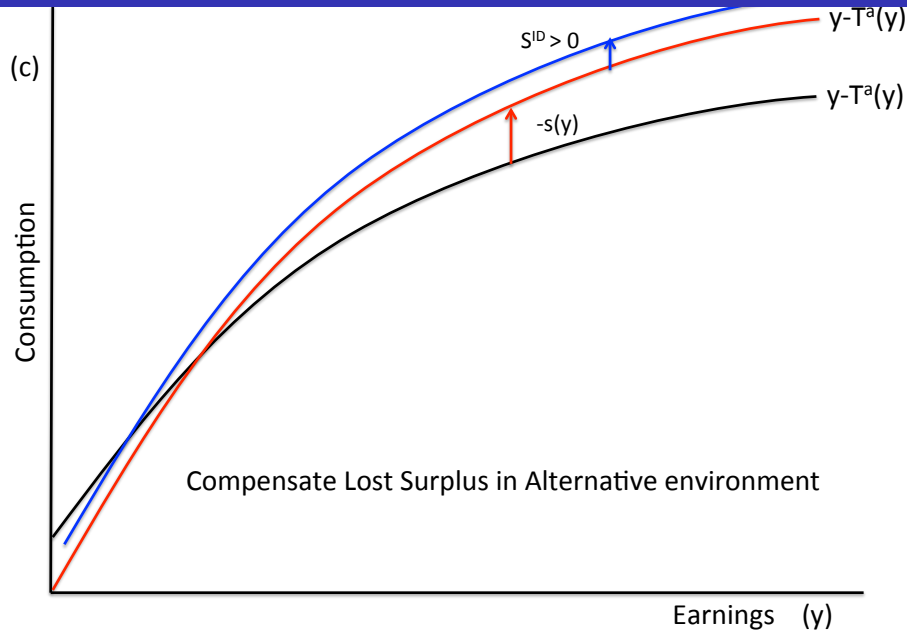
"CV" Example



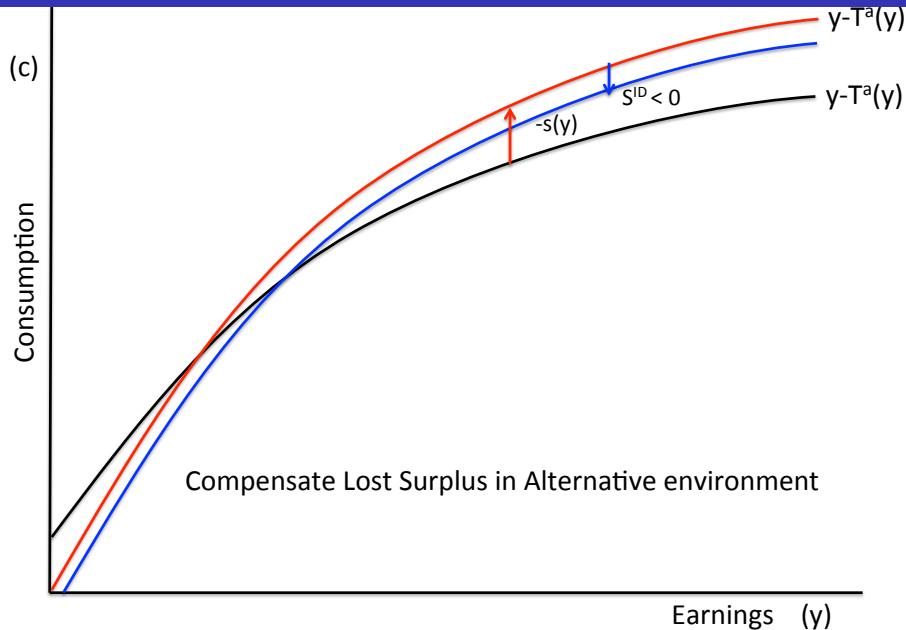
"CV" Example



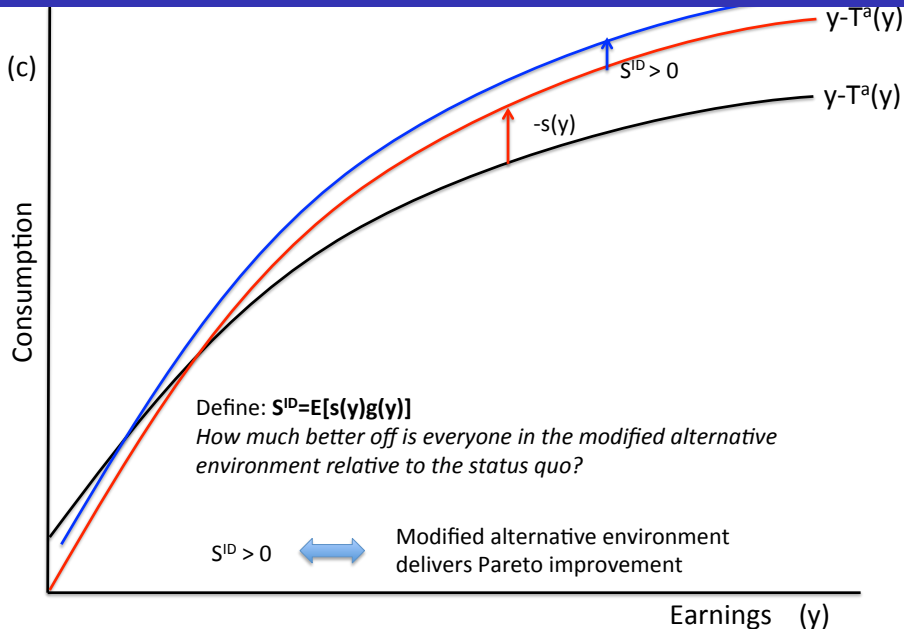
"CV" Example



"CV" Example



"CV" Example



- If $g(y)$ is similar in status quo and alternative environment, then EV and CV are first-order equivalent
 - Proof?
- When surplus is homogeneous conditional on income:
 - S^{ID} provides first-order characterization of potential Pareto comparisons
 - S^{ID} quantifies difference between environments without making inter-personal comparisons
 - By how much is *everyone* better off?
 - What if surplus is heterogeneous conditional on income?

Estimating the Marginal Cost of Taxation

- What do we need to estimate $FE(y)$?
- A bunch of exogenous variation in the tax schedule
 - Combined with data on government revenue, q
 - Then, compute

$$1 + FE(y) = \lim_{\epsilon \rightarrow 0} \frac{\partial \hat{q}(y, \epsilon, \eta)}{\partial \eta}$$

- But, need tax variation separate for each y !
 - In practice: look at responses to policy changes + add a bit of structure

Recall: Behavioral Responses to Tax Changes

- Existing evidence on behavioral responses to taxation provides guidance on $1 + FE(y)$
 - EITC causes people to:
 - Enter the labor force (summary in Hotz and Scholz (2003))
 - Distort earnings (Chetty et al 2013).
 - $1 + FE(y) \approx 1.14$ for low-earners (calculation in Hendren 2013)
 - Taxing top incomes causes:
 - Reduction in taxable income (review in Saez et al 2012)
 - Implies $1 + FE(y) \approx 0.50 - 0.75$
 - Disagreement about amount, but general agreement on the sign:
 $FE(y) < 0$
- Reduced form empirical evidence suggests should put more weight on surplus to poor
 - Despite evidence that taxable income elasticities may be quite stable across the income distribution (e.g. Chetty 2012)

A More Precise Representation

- Use optimal tax approach to write $FE(y)$ as function of taxable income elasticities
- Let

$\epsilon^c(y) =$ avg comp. elasticity for those earning y

$\zeta(y) =$ avg inc. effect for those earning y

$\epsilon^P(y) =$ avg LFP rate elasticity for those earning y

Optimal Tax Expression

For every point, y^* , such that $T'(y)$ and $\epsilon^c(y^*)$ are locally constant and the distribution of income is continuous:

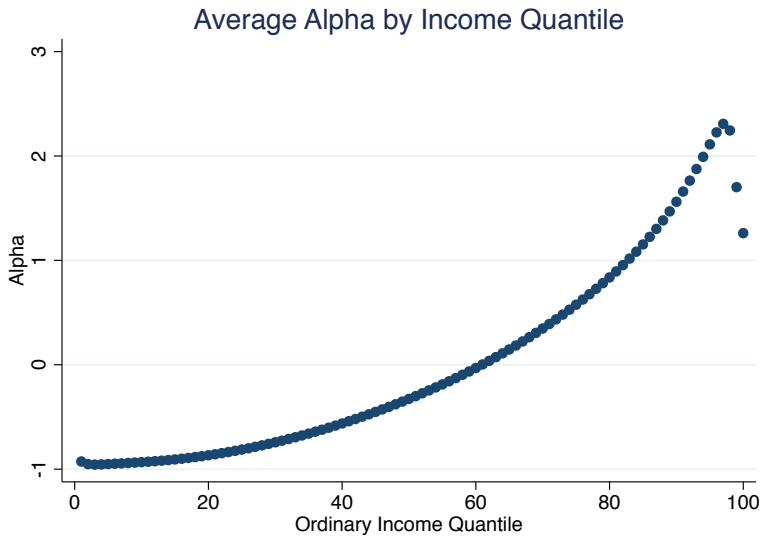
$$FE(y^*) = \underbrace{-\epsilon^P(y^*) \frac{T(y) - T(0)}{y - T(y)}}_{\text{Participation Effect}} - \underbrace{\zeta(y^*) \frac{\tau(y^*)}{1 - \frac{T(y^*)}{y^*}}}_{\text{Income Effect}} - \underbrace{\epsilon^c(y^*) \frac{\tau(y^*)}{1 - \tau(y^*)} \alpha(y^*)}_{\text{Substitution Effect}}$$

where $\alpha(y) = -\left(1 + \frac{yf'(y)}{f(y)}\right)$ is the local Pareto parameter of the income distribution

- Heterogeneity in $FE(y)$ depends on:
 - 1 Shape of income distribution, $\alpha(y)$
 - 2 Shape and size of behavioral elasticities
 - 3 Shape of tax rates
- See derivation in Bourguignon and Spadaro (2012), Zoutman (2013a, 2013b), and Hendren (2014)

Estimation Approach in US (Hendren, 2014)

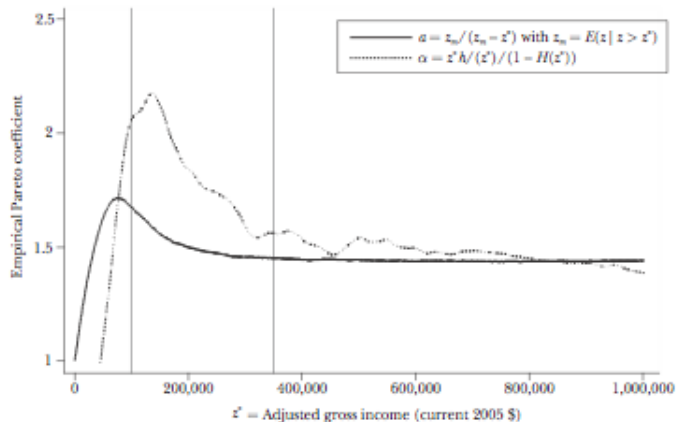
- Calibrate behavioral elasticities from existing literature on taxable income elasticities
 - Assess robustness to range of estimates (e.g. compensated elasticity of 0.1, 0.3, and 0.5)
- Estimate shape of income distribution and marginal income tax rate using universe of US income tax returns
 - Account for covariance between elasticity of income distribution and marginal tax rate



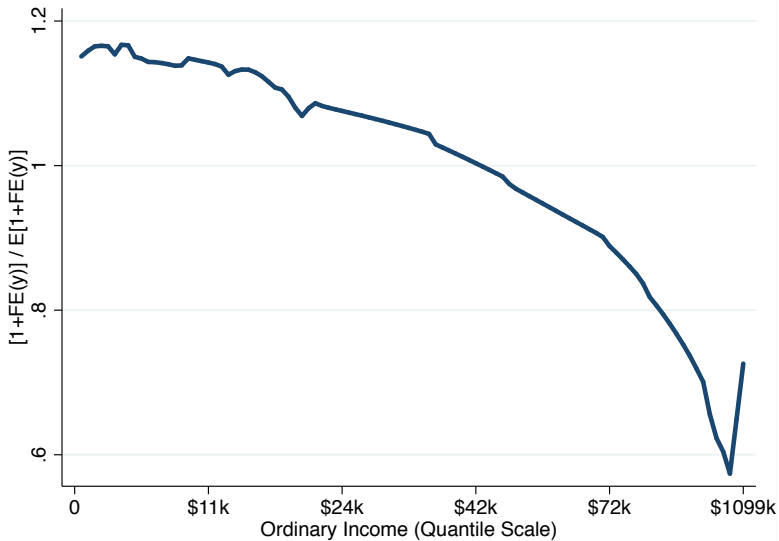
Pareto Tails

Figure 2

Empirical Pareto Coefficients in the United States, 2005



Shape of $1+FE(y)$



Example: Producer versus Consumer Surplus

- Suppose budget neutral policy with benefits to producers S^P and consumers S^C
 - Extreme assumption: producer surplus falls to top 1%
 - Consumer surplus falls evenly across income distribution

- Optimal weighting:

$$S^{ID} = 0.77S^P + S^C$$

- “Consumer surplus standard” requires top tax rate near Laffer curve
 - France should have tighter merger regulations?
- Key assumption: policy is budget neutral (inclusive of fiscal externalities)
- What about non-budget neutral policies?

Targeted Policies

- Suppose G affects those with income y
- Construct

$$MVPF_G = \frac{s(y)}{1 + FE^G}$$

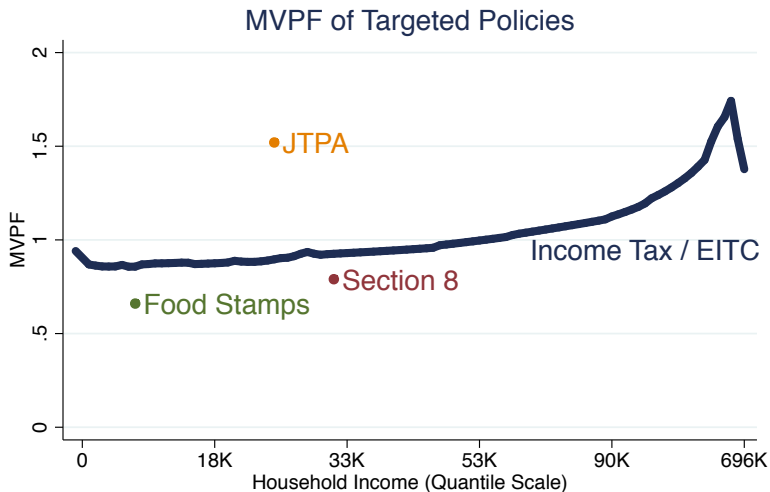
- Depends on causal effects (FE^G) and WTP for non-market good
- Additional spending on G desirable iff

$$\underbrace{MVPF_G}_{\text{Value of } G} \geq \frac{1}{\underbrace{1 + FE(y)}_{\text{Value of } T(y)}}$$

- Compare value of spending to value of equivalent tax cut to similar people

Policy	$\frac{\partial u}{\partial G}$ λ	$\frac{1}{\int_i} \int_i \frac{d\hat{t}_i}{d\theta} di$	<i>MVPF</i>
Taxes (Top Tax Rate)	1	1.33 - 2	1.33 - 2
Taxes (EITC Expansion)	1	0.88	0.88
Food Stamps	0.8 - 1	0.66	0.53 - 0.66
Job Training	0 - 1.22	1.52	0 - 1.85
Housing Vouchers	0.83	0.95	0.79

- Map to average incomes associated with each policy



Source: MVPFs compiled in Hendren (2013) drawing on existing estimates from Bloom et al (1997), Hoynes and Schanzenbach (2012), and Jacob and Ludwig (2012)
Income is average income of policy beneficiaries normalized to 2012 income using CPI-U

Two reasons to use these weights...

- Logic: Compare the value of the policy to a tax cut with similar distributional incidence
 - Can augment policy with benefit tax to make Pareto improvement
 - Prefer the policy by the (potential) Pareto principle
- Rationales not to use these weights?
 - e.g. Political economy constraints on redistribution? Others?

Inverse Optimum Approach

- Up to now, $1 + FE(y)$ is the **cost** of taxation
 - Not necessarily a normative value aside from being able to search for potential Pareto improvements
- But, can also have a normative interpretation
 - Reveals the social preferences of whoever set the tax schedule
 - “Inverts” the optimal tax idea

Optimal Tax: Social Preferences \implies Taxes

”Inverse-Optimal” Tax: Taxes \implies Social Preferences

Inverse Optimum Derivation

- Social welfare:

$$W = \int v(\theta) \psi(\theta) d\mu(\theta)$$

- Define social welfare $\hat{W}(y^*, \epsilon, \eta)$ to be social welfare under $\hat{T}(y; y^*, \epsilon, \eta)$
- Let $v(\theta)$ denote the social marginal utility of income for type θ :

$$v(\theta) = \frac{dW}{dy_\theta} = \lambda(\theta) \psi(\theta)$$

where λ is the individual's marginal utility of income

- So, v is the impact on social welfare of giving type θ an additional \$1.
 - Ratios of v are Okun's bucket (Okun (1975))

$$\frac{v(\theta_1)}{v(\theta_2)} = 2$$

implies indifferent to \$1 to type θ_1 relative to \$2 to type θ_2

Inverse Optimum Derivation

- For a given y^* , what is the welfare impact of increasing transfers to those earning near y by η ?
- Use envelope theorem:
 - Marginal welfare impact given by mechanical loss in income weighted by social marginal utility of income:

$$\frac{dW}{d\eta}\Big|_{\eta=0} = \int v(\theta) \mathbf{1}\left\{y \in \left(y^* - \frac{\epsilon}{2}, y^* + \frac{\epsilon}{2}\right)\right\} d\mu(\theta)$$

- Note: Assumes partial equilibrium
- So, a localized tax cut yields welfare:

$$\lim_{\epsilon \rightarrow 0} \frac{dW}{d\eta}\Big|_{\eta=0} = E[v(\theta) | y(\theta) = y^*]$$

or, the average marginal utilities of income for those earning y^*

Inverse Optimum Derivation

- Government is indifferent to tax changes if and only if

$$E [v(\theta) | y(\theta) = y^*] = 1 + FE(y^*) \quad \forall y^*$$

- Exercise: Show this is equivalent to equating all MVPFs associated with tax changes to each other
- Common simplifying assumption: Unidimensional heterogeneity:

$$E [v(\theta) | y(\theta) = y] = v(y)$$

- Otherwise reveals average social marginal utilities of income conditional on income

Inverse Optimum Implementation

- Implement using common elasticity representation
 - Assume convex preferences (no participation responses) and no income effects
- Recall that if $\tau(y)$ is linear then

$$g(y^*) = 1 + \epsilon \frac{\tau(y)}{1 - \tau(y)} \frac{d}{dy} \Big|_{y=y^*} \left[y \frac{f(y)}{f(y^*)} \right]$$

where $\frac{d}{dy} \Big|_{y=y^*} \left[y \frac{f(y)}{f(y^*)} \right] = - \left(1 + \frac{y^* f'(y^*)}{f(y^*)} \right)$ is the local Pareto parameter of the income distribution

- But, if τ is nonlinear, this generalizes to:

$$g(y^*) = 1 + \epsilon \frac{d}{dy} \Big|_{y=y^*} \left[\frac{\tau(y)}{1 - \tau(y)} y \frac{f(y)}{f(y^*)} \right]$$

or

$$\frac{g(y^*) - 1}{\epsilon} f(y^*) = \frac{d}{dy} \Big|_{y=y^*} \left[\frac{\tau(y)}{1 - \tau(y)} y f(y) \right]$$

- Use Fundamental Thm of Calculus:

$$\left[\lim_{\tilde{y} \rightarrow \infty} \frac{\tau(y)}{1 - \tau(y)} \frac{yf(y)}{f(y^*)} \right] - \frac{\tau(y)}{1 - \tau(y)} yf(y) = \int_y^\infty \frac{g(\tilde{y}) - 1}{\epsilon} f(\tilde{y}) d\tilde{y}$$

- Generally, $\lim_{\tilde{y} \rightarrow \infty} \frac{\tau(y)}{1 - \tau(y)} \frac{yf(y)}{f(y^*)} = 0$ (e.g. if f is pareto, $f \propto y^{-\alpha-1}$)

- So

$$\frac{\tau(y)}{1 - \tau(y)} yf(y) = \int_y^\infty \frac{1 - g(\tilde{y})}{\epsilon} f(\tilde{y}) d\tilde{y}$$

- Implies basic Mirrlees formula:

$$\frac{\tau(y)}{1 - \tau(y)} \alpha(y) \epsilon(y) = 1 - G(y)$$

where

$$G(y) = \frac{1}{1 - F(y)} \int_y^{\infty} g(\tilde{y}) f(\tilde{y}) d\tilde{y}$$

is the average social marginal utilities on those earning more than y

$$\alpha(y) = \frac{yf(y)}{1 - F(y)}$$

is the local Pareto parameter of the income distribution

- Growing literature estimating inverse optimum solutions in many settings
 - Bourguignon and Spadaro (2012), Jacobs, Jongen, and Zoutman (2013; 2014), Lockwood and Weinzierl (2014),
- Key inputs:
 - Tax schedule, $\tau(y)$
 - Shape of income distribution, $\alpha(y)$
 - Taxable income elasticity, $\epsilon(y)$
- Key Assumptions
 - constant elasticity (consensus that $\epsilon = 0.5$?)
 - no other responses (e.g. participation)

- Bourguignon and Spadaro (2012) were one of the first to empirically implement the inverse optimum approach
- Use survey data in France
- Use wages as y
 - Problems with this?
 - Recall: Census/survey data vs. tax data...Piketty and Saez (2003)

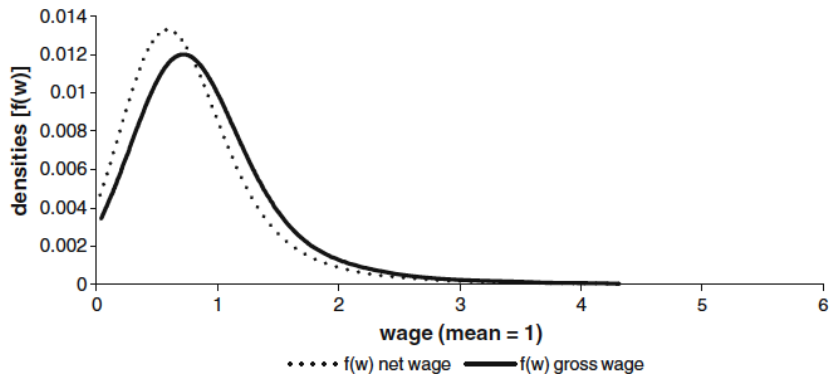


Fig. 2 Kernel wage densities for singles: net and gross scenario

- Problem: tax rates vary conditional on wage
- Ideally, estimate tax rate separately using tax data
 - Then aggregate the fiscal externality (see Hendren 2016)
- Solution in survey data: smooth it...

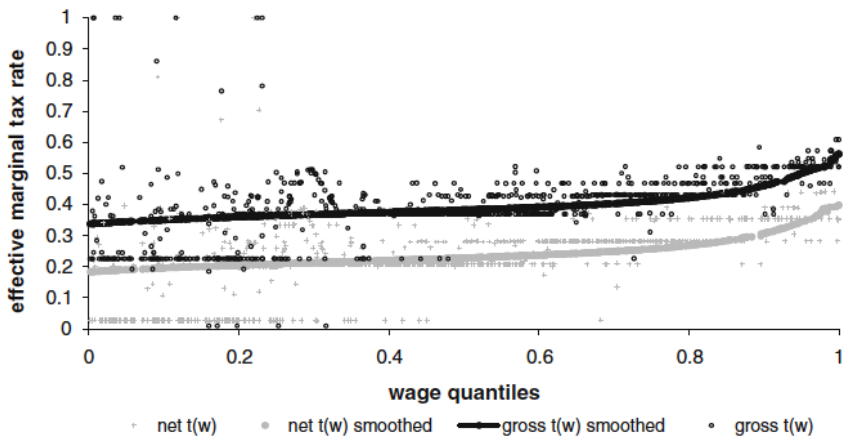


Fig. 1 Kernel smoothed marginal tax rates for singles: net and gross scenarios

- Finally, need elasticity of taxable income
- Use range of elasticities between 0.1 and 0.5

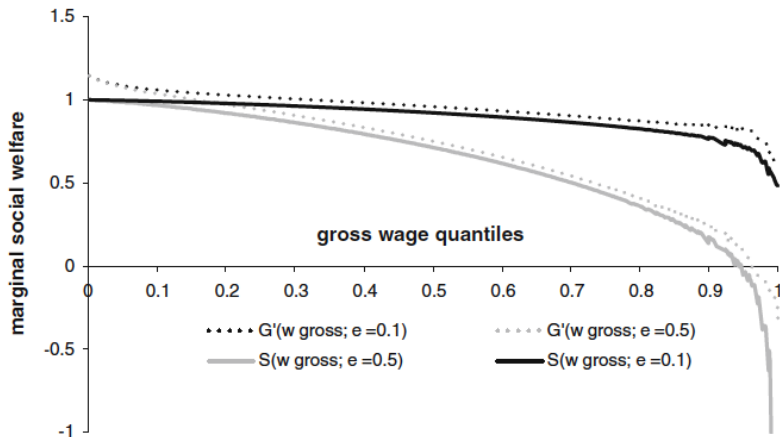


Fig. 4 Social marginal welfare for singles (on gross wages)

- Conclusion: If taxable income elasticity is 0.5, then taxes on the rich are too high:

$$FE(y) < -1$$

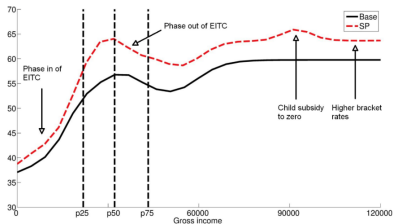
- Above the top of the laffer curve
- Potential limitations?
- Recall from optimal tax:
 - optimal for top tax rate to be zero unless we have thick upper tail of income distribution
- Nathan's take: result largely driven by thin tail of income distribution provided in survey data; unclear whether would hold in tax data

- Jacobs, Jongen, and Zoutman (2016) “Redistributive Politics and the Tyranny of the Middle Class”
- Key idea: Use not only equilibrium tax rates, but proposed tax changes to estimate social preferences
 - Use data from Dutch political parties
- Map variations in tax policies, $\tau^j(y)$, into implied social welfare weights, $G^j(y)$
 - Infer political preferences of parties

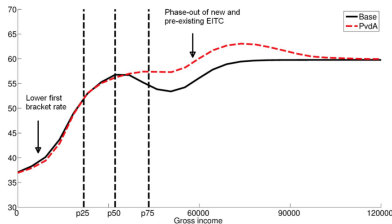
$$\frac{\tau^j(y)}{1 - \tau^j(y)} \alpha(y) \epsilon(y) = 1 - G^j(y)$$

where G^j is the implied social welfare weights on those earning more than y

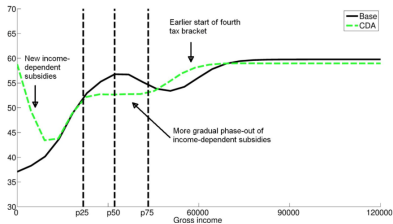
Effective Marginal Tax Rates



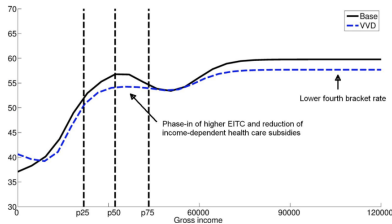
(a) Socialist Party (SP)



(b) Labor Party (PvdA)

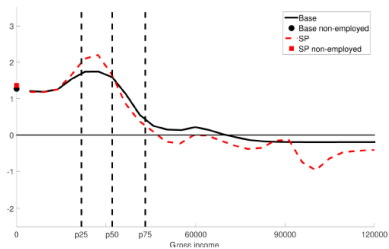


(c) Christian-Democratic Appeal (CDA)

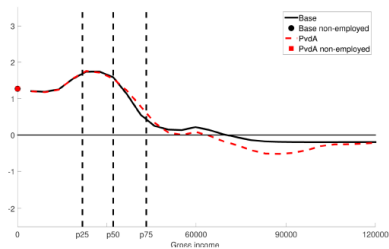


(d) People's Party for Freedom and Democr. (VVD)

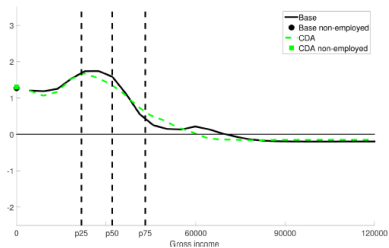
Implied Social Preferences



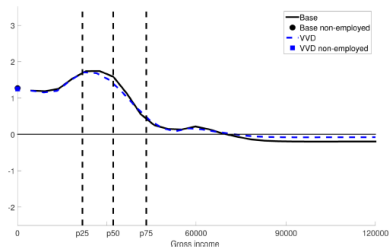
(a) Socialist Party (SP)



(b) Labor Party (PvdA)



(c) Christian-Democratic Appeal (CDA)



(d) People's Party for Freedom and Democr. (VVD)

- Potential concerns:
 - Dynamics of policy responses
 - Changes in income distribution
 - Changes in taxable income elasticity
- General issue with “sufficient statistics”?

- Redistribution isn't free
 - Empirical evidence suggests it is costly (cheap) to redistribute from rich to poor (poor to rich)
 - Policies targeted towards the poor “should” be inefficient relative to a world with lump-sum transfers
- But, accounting for distributional incidence requires estimating fiscal externalities
 - Taxable income elasticity is a tough empirical parameter...
 - And, still need to estimate MVPF of a policy (which requires estimating its fiscal externality too...)
- Can we reduce these requirements of estimating all these behavioral responses?
 - Next lecture!

Heterogeneous Surplus

- What if policy affects different types conditional on income?
 - e.g. Medicaid affects the poor and sick; EITC affects the poor and healthy
 - And maybe there's a social preference for the sick conditional on income?
- Redistribution based on income, not individual-specific
 - Two people with same income, $y(\theta)$, can have different surplus, $s(\theta)$
 - Income tax is a “blunt instrument”
 - $\int s(\theta) g(y(\theta)) =$ how much *on average* is each income level better off
 - Search for potential Pareto comparisons more difficult

Heterogeneous Surplus

- Option 1: Still can search for potential Pareto improvements
- Define

$$\underline{S}^{ID} = E [\min \{s(\theta) | y(\theta) = y\} g(y)] > 0$$

- Modified alternative environment delivers Pareto improvement iff $\underline{S}^{ID} > 0$
- Modified status quo offers Pareto improvement iff $\bar{S}^{ID} < 0$
- No potential Pareto ranking when $\underline{S}^{ID} < 0 < \bar{S}^{ID}$
- Easier if surplus does not vary conditional on income, so that $\underline{S}^{ID} = S^{ID} = \bar{S}^{ID}$

Generalization to multiple dimensions

- Option 2: Add more status quo policies
- Marginal cost $1 + FE(\mathbf{X})$ as opposed to $1 + FE(y)$
 - e.g. Transfers conditional on both income, y , and medical spending, m ;
 - Notation: $\mathbf{X} = \{y, m\}$
- How do we construct $FE(\mathbf{X})$?
- Construct $FE(\mathbf{X}) = \lim_{\epsilon \rightarrow 0} FE(\mathbf{X}, \epsilon)$, where $FE(\mathbf{X}^*, \epsilon) = \frac{d}{d\eta} q(\mathbf{X}, \eta, \epsilon) - 1$ is the fiscal externality from giving a tax cut to those with values of $\mathbf{X} \in N_\epsilon(\mathbf{X}^*)$
 - $q(\mathbf{X}, \eta, \epsilon)$ is government revenue when types within an ϵ -neighborhood of \mathbf{X} obtain a tax cut of η
- Then, test

$$\int [1 + FE(\mathbf{X})] s(\mathbf{X}) >? 0$$