Modeling Insurance Markets

Nathaniel Hendren

Harvard

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Modeling Competition Insurance Markets is Tough

- There is no well-agreed upon model of competitive insurance markets
  - Despite 50 years of research!
- Standard notions of pure strategy competitive equilibria break down
  - Preferences/Demand are related to cost
- Insurers can manipulate not only price but also the design of contracts to affect their own (and others) costs
  - Leads to unraveling!

Foundational Literature: Akerlof 1970 & RS76

- Akerlof (1970): Cars lose value the day after they're sold...
  - Argued that market for health insurance above age 65 does not exist because of adverse selection
    - Market unraveled because of adverse selection "death spiral"
- Problem with model: single contract traded, so competition only on price
  - Rothschild and Stiglitz (1976) + 1000+ other papers...
    - Compete on more than 1 dimension of the contract
    - Standard game-theoretic notions of (pure strategy) equilibria may not exist -> "Market unraveling"

Outline

- Clarify when the standard competitive model goes wrong (and hence we have to choose amongst competing game-theoretic models)
  - Clarify what we mean by "unraveling"
- Discuss 2 classes of "solutions" to non-existence
  - Miyazaki-Wilson-Spence (Reach the constrained pareto frontier)
  - Riley (1979) (Don't reach the frontier)
- Context: Binary insurance model with uni-dimensional type distribution

Model

Model Environment

- Unit mass of agents endowed with wealth w
- Face potential loss of size l with privately known probability p
  - Distributed with c.d.f. $F(p)$ with support Ψ
    - Could be continuous, discrete or mixed
    - Rothschild and Stiglitz (1976): $p \in \{p_L, p_H\}$ (2 types)
  - Let P denote random draw from population (c.d.f. $F(p)$)
- Agents vNM preferences

$$pu(c_L) + (1-p)u(c_{NL})$$

Insurers / timing

- Insurance structure: Rothschild and Stiglitz (1976) with menus
- There exists a set of risk-neutral insurance companies, $j \in J$ seeking to maximize expected profits by choosing a menu of consumption bundles:

$$A_j = \left\{ c_L^j (p), c_{NL}^j (p) \right\}_{p \in \Psi}$$

- First, insurers simultaneously offer a menu of consumption bundles
- Given the set of available consumption bundles,

$$A = \cup_j A_j$$

individuals choose the bundle that maximizes their utility

Equilibrium

Definition

An allocation $A = \{c_L(p), c_{NL}(p)\}_{p \in \Psi}$ is a **Competitive Nash Equilibrium** if

1. A is incentive compatible

$$pu(c_L(p)) + (1-p)u(c_{NL}(p)) \geq pu(c_L(\tilde{p})) + (1-p)u(c_{NL}(\tilde{p})) \quad \forall p, \tilde{p}$$

2. A is individually rational

$$pu(c_L(p)) + (1-p)u(c_{NL}(p)) \geq pu(w-l) + (1-p)u(w) \quad \forall p \in \Psi$$

3. A has no profitable deviations [Next Slide]

No Profitable Deviations

For any other menu, $\hat{A} = \left\{\hat{c}_L\left(p\right), \hat{c}_{NL}\left(p\right)\right\}_{p \in \Psi}$, it must be that

$$\int_{p \in D\left(\hat{A}\right)} \left[p\left(w - l - c_L\left(p\right)\right) + \left(1 - p\right)\left(w - c_{NL}\left(p\right)\right)\right] dF\left(p\right) \leq 0$$

where

$$D\left(\hat{A}\right) = \left\{ p \in \Psi | \begin{array}{c} \max_{\hat{p}} \left\{pu\left(\hat{c}_L\left(\hat{p}\right)\right) + \left(1 - p\right) u\left(\hat{c}_{NL}\left(\hat{p}\right)\right)\right\} \\ > \\ pu\left(c_L\left(p\right)\right) + \left(1 - p\right) u\left(c_{NL}\left(p\right)\right) \end{array} \right\}$$

- $D\left(\hat{A}\right)$ is the set of people attracted to $\hat{A}$
- Require that the profits earned from these people are non-positive

Two Definitions of Unraveling

- **Akerlof unraveling**
  - Occurs when demand curve falls everywhere below the average cost curve
  - Market unravels and no one gets insurance
- **Rothschild and Stiglitz unraveling**
  - Realize a Competitive Nash Equilibrium may not exist
  - Market unravels a la Rothschild and Stiglitz when there does not exist a Competitive Nash Equilibrium

Akerlof Unraveling

Theorem

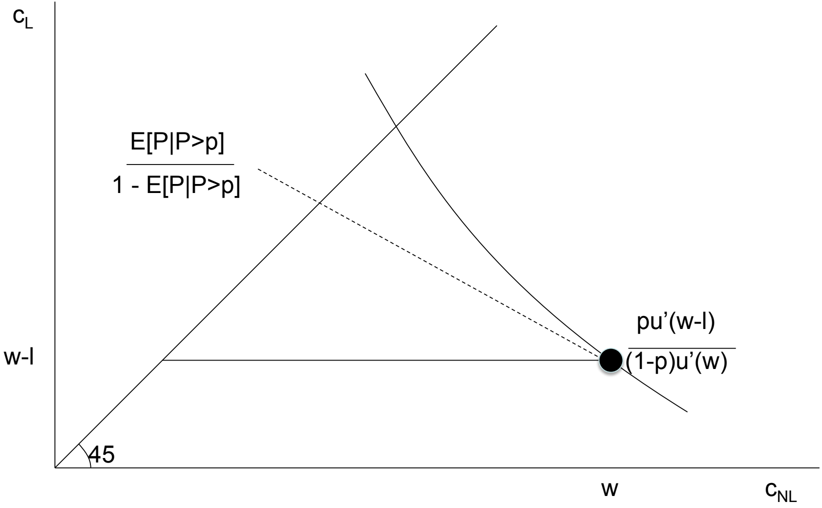
The endowment, $\{(w-l,w)\}$, is a competitive equilibrium if and only if

$$\frac{p}{1-p}\frac{u'(w-l)}{u'(w)} \leq \frac{E[P|P\geq p]}{1-E[P|P\geq p]}\ \forall p\in\Psi\backslash\{1\} \qquad (1)$$

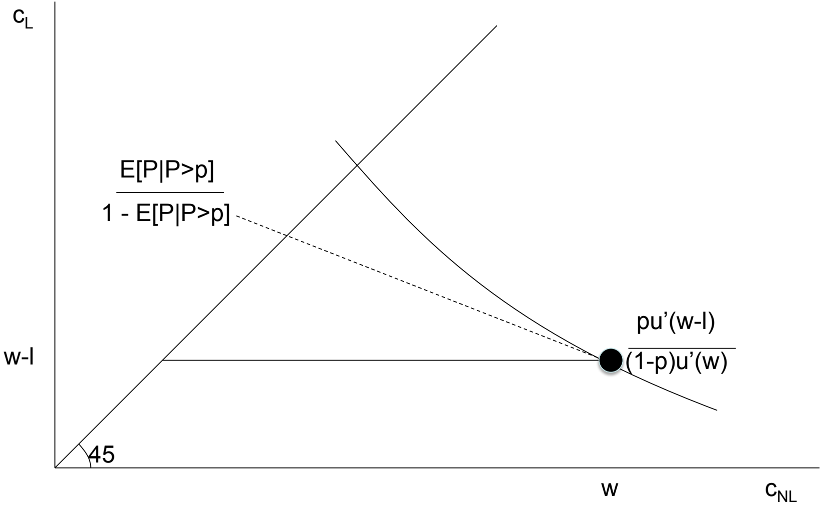
where $\Psi\backslash\{1\}$ denotes the support of $F(p)$ excluding the point $p=1$.

- The market unravels a la Akerlof when no one is willing to pay the pooled cost of worse risks (Hendren 2013)
  - Theorem extends Akerlof unraveling to set of all potential traded contracts, as opposed to single contract
  - No gains to trade -> no profitable deviations by insurance companies

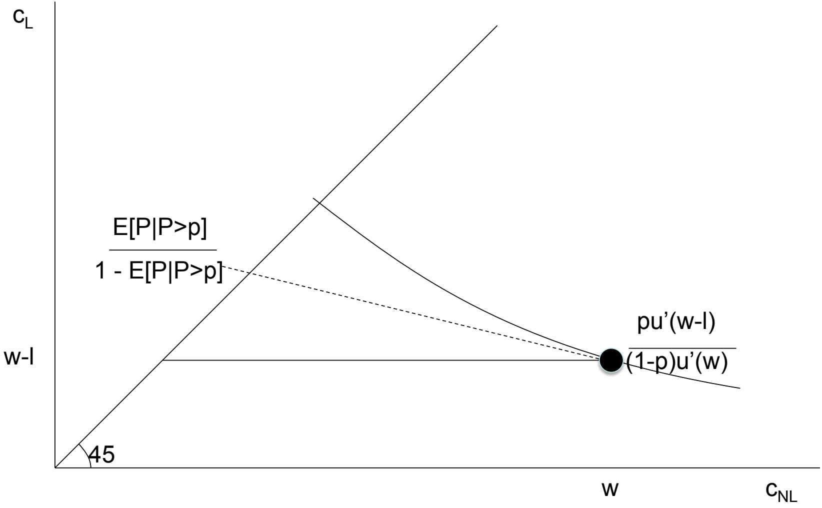
Akerlof Unraveling



Akerlof Unraveling (2)



Akerlof Unraveling (3)



Aside: High Risks

- Corollary: If the market fully unravels a la Akerlof, there must exist arbitrarily high risks:

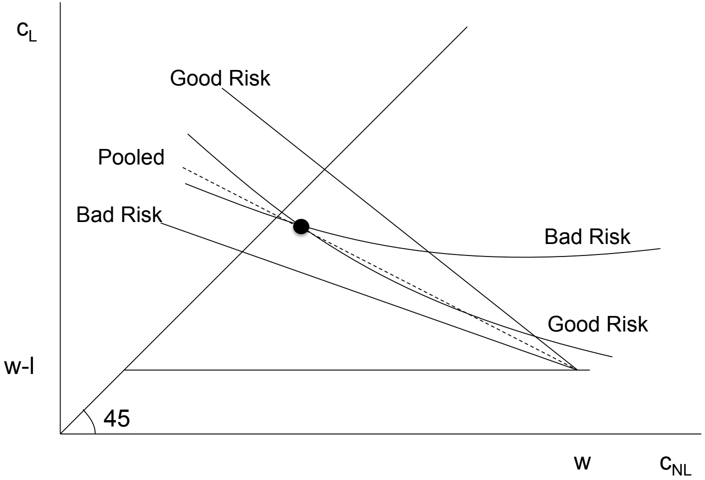
$$F(p) < 1 \quad \forall p < 1$$

- Need full support of type distribution to get complete Akerlof unraveling
  - Can be relaxed with some transactions costs (see Chade and Schlee, 2013)

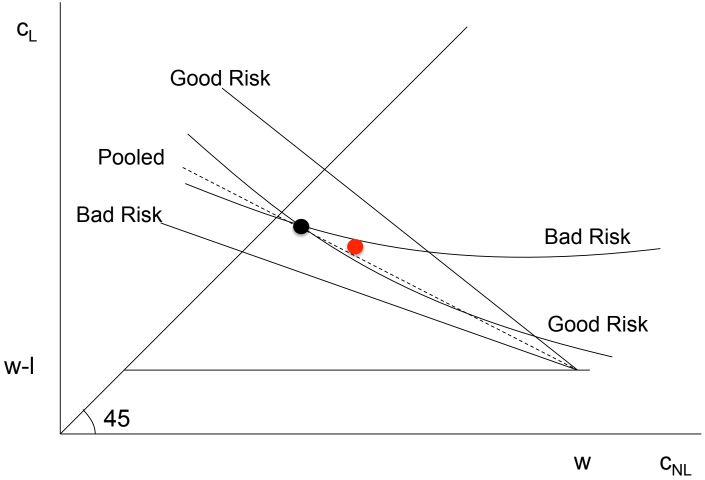
Rothschild and Stiglitz Unraveling

- When does a Competitive Nash Equilibrium exist?
- Here, I follow Rothschild and Stiglitz (1976) and Riley (1979)
- Generic fact: Competition -> zero profits
- Key insight of Rothschild and Stiglitz (1976): Nash equilibriums can't sustain pooling of types

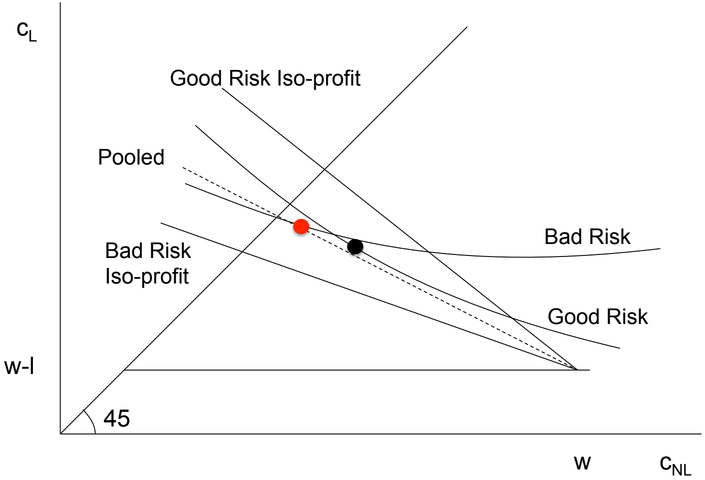
Rothschild and Stiglitz: No Pooling



Rothschild and Stiglitz: No Pooling (2)



Rothschild and Stiglitz: No Pooling (3)



Regularity condition

- No pooling + zero profits -> No cross subsidization:

$$p c_L(p) + (1-p) c_{NL}(p) = w - pl \quad \forall p \in \Psi$$

  - Insurers earn zero profits on each type
- A Regularity Condition
- Suppose that either:
  1. There exists an interval over which P has a continuous distribution
  2. $P = 1$ occurs with positive probability
- Satisfied if either F is continuous or F is discrete with $p = 1$ in the support of the distribution
- Can approximate any distribution with distributions satisfying the regularity condition

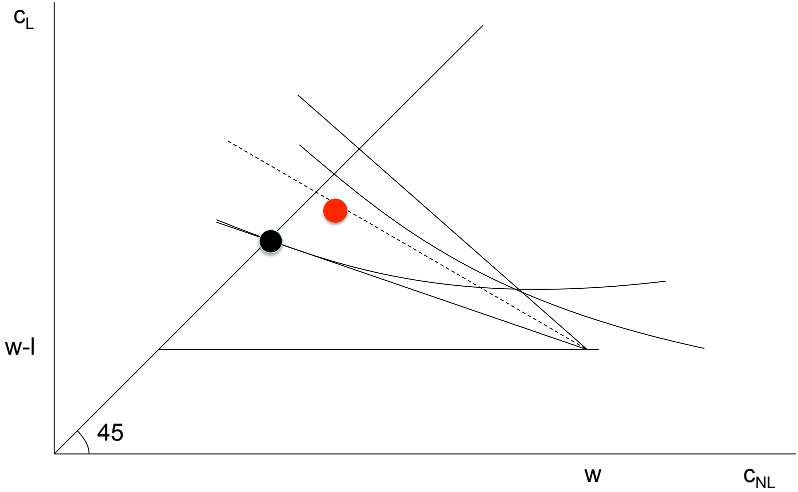
Result #2: Exhaustive of Possible Occurances

Theorem

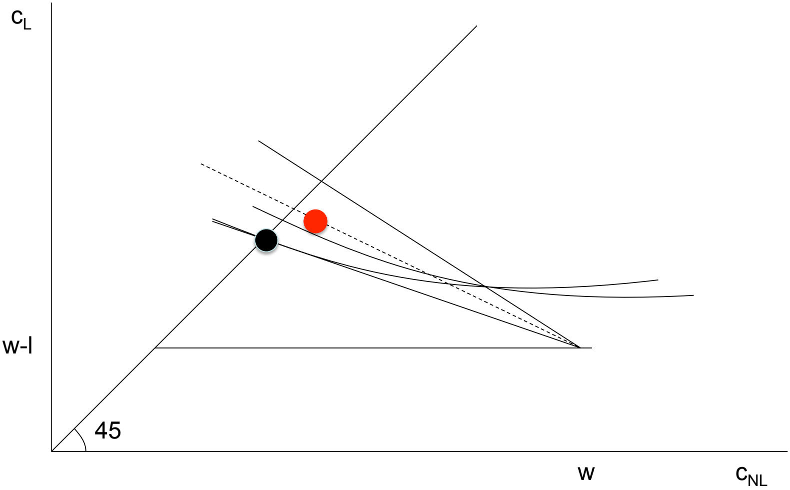
Suppose the regularity condition holds. Then, there exists a Competitive Nash Equilibrium if and only if the market unravels a la Akerlof (1970)

- Either no one is willing to cross-subsidize -> no profitable deviations that provide insurance
- Or, people are willing to cross-subsidize -> generically, this can't be sustained as a Competitive Nash Equilibrium
- Proof: Need to show that Nash equilibrium does not exist when Akerlof unraveling condition does not hold
  - Case 1: $P = 1$ has positive probability
    - Risks $p < 1$ need to subsidize $p = 1$ type in order to get insurance
  - Case 2: P is continuous and bounded away from $P = 1$.
    - We know Akerlof unraveling condition cannot hold
    - Follow Riley (1979) – shows there's an incentive to pool types -> breaks potential for Nash equilibrium existence

Generic No Equilibrium (Riley)



Generic No Equilibrium (Riley) (2)



Summary

- Generically, either the market unravels a la Akerlof or Rothschild and Stiglitz
- No gains to trade -> unravels a la Akerlof
  - No profitable deviations -> competitive equilibrium exists
- Gains to trade -> no unraveling a la Akerlof
  - But there are profitable deviations
  - Generically, no Competitive Equilibrium (unravels a la Rothschild and Stiglitz)
- We don't have a model of insurance markets!
  - Generically, the standard Nash model generically fails to make predictions precisely when there are theoretical gains to trade

Solutions to the Non-Existence Problem

- Two classes of models in response to non-existence
- Consider 2-stage games:
- Stage 1: firms post menu of contracts
- Stage 2: Assumption depends on equilibrium notion:
  - Miyazaki-Wilson-Spence: Firms can drop unprofitable contracts
    - Formalized as dynamic game in Netzer and Scheuer (2013)
  - Riley: Firms can add contracts
    - Formalized as dynamic game in Mimra and Wambach (2011)
- Then, individuals choose insurance contracts

MWS

- Miyazaki (1979); Wilson (1977); Spence (1978)
- Two Stage Game:
  - Firms choose contracts
    - Menus (Miyazaki)
    - Single contracts (Wilson / Spence)
  - Firms observe other contracts and can drop (but not add) contracts/menus
    - In Miyazaki, firms have to drop the entire menu
  - Individuals choose insurance from remaining set of contracts

MWS

- Reaching the Pareto frontier requires allowing some contracts to run deficits/surplus
  - Individuals generically are willing to "buy off" worse risks' incentive constraints
- Miyazaki Wilson Spence allows for this if the good types want to subsidize the bad types
  - If you try to steal my profitable contract, I drop the corresponding negative profit contract and you get dumped on!
- **MWS equilibrium maximizes welfare of best risk type by making suitable compensations to all other risk types to relax IC constriant**
  - Fully separating solution in Miyazaki
  - Can be pooling in Wilson / Spence

Riley (1979)

- Predicts "fully separating" contracts with no cross-subsidization across types
  - IC constraint + zero profit constraints determine equilibrium
- Why no cross-subsidization?
  - If cross-subsidization, then firms can add contracts.
  - But, firms forecast this response and therefore no one offers these subsidizing contracts
- Predicts no trade if full support type distribution

Other non-game-theoretic approaches

- Walrasian:
  - Bisin and Gotardi (2006)
    - Allow for trading of choice externalities -> reach efficient frontier/MWS equilibrium (pretty unrealistic setup...)
  - Azevedo and Gottlieb (2015) -> reach inefficient Riley equilibria
- Search / limited capacity / limited liability / cooperative solutions / etc.
  - Guerrieri and Shimer (2010) -> reach inefficient Riley equilibria

Empirical Question?

- Need theory of a mapping from type distributions to outcomes
  - Standard model works if prediction is no trade
    - This happens for those with “pre-existing conditions” in LTC, life, and disability insurance (Hendren 2013)
  - But, standard model fails when market desires cross-subsidization
    - Key debate: can competition deliver cross-subsidization?
    - Should be empirical question!?
- In short, insurance markets are fun because no one agrees about how to model them!